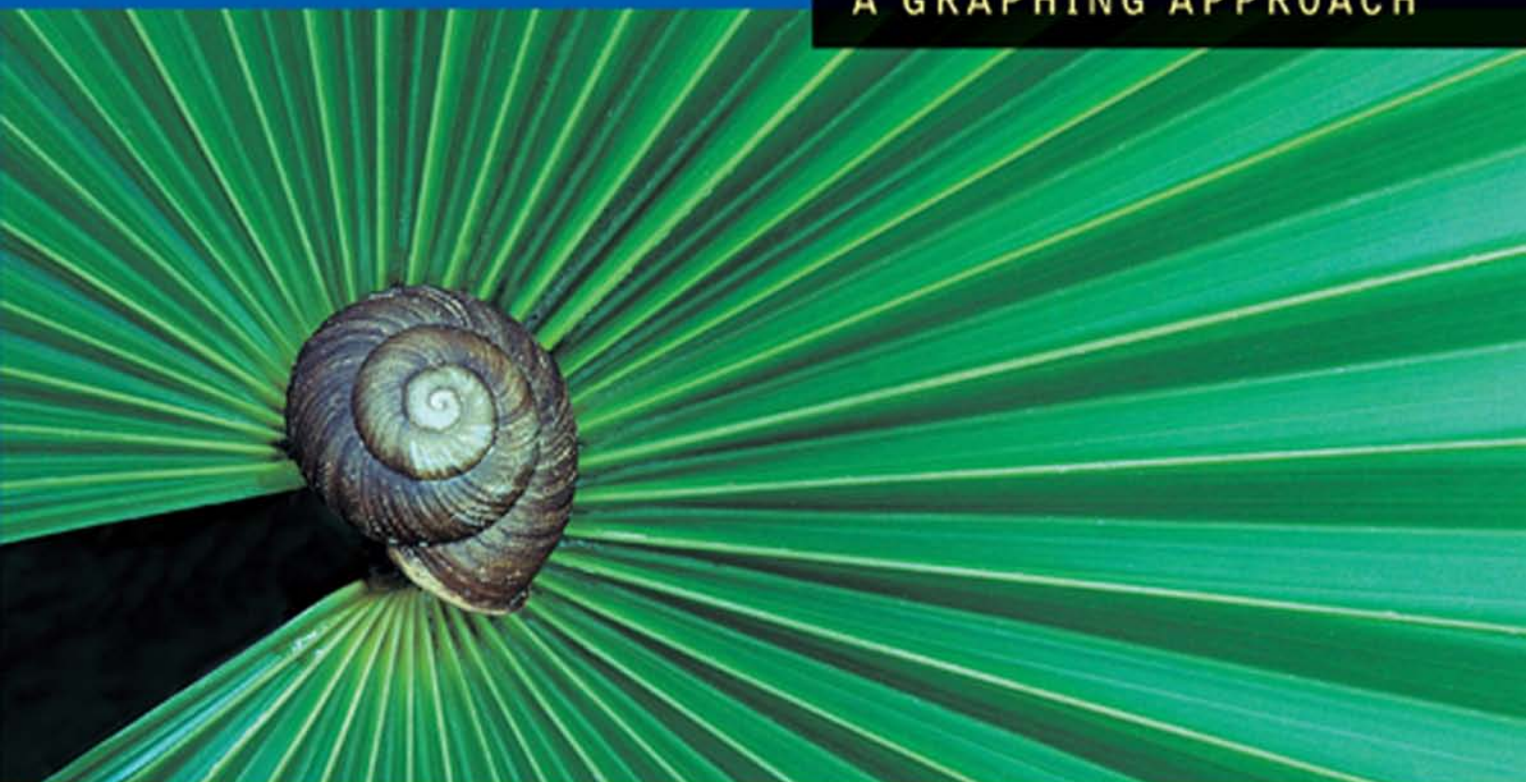
LARSON HOSTETLER EDWARDS

COLLEGE ALGEBRA

A GRAPHING APPROACH



FIFTH EDITION

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College Algebra
A Graphing Approach

Ron Larson
The Pennsylvania State University
The Behrend College

Robert Hostetler
The Pennsylvania State University
The Behrend College

Bruce H. Edwards
University of Florida

With the assistance of David C. Falvo
The Pennsylvania State University
The Behrend College

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Sponsoring Editor: Cathy Cantin
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Contents

A Word from the Authors

Welcome to *College Algebra: A Graphing Approach*, Fifth Edition. We are pleased to present this new edition of our textbook in which we focus on making the mathematics accessible, supporting student success, and offering instructors flexible teaching options.

Accessible to Students

We have taken care to write this text with the student in mind. Paying careful attention to the presentation, we use precise mathematical language and a clear writing style to develop an effective learning tool. We believe that every student can learn mathematics, and we are committed to providing a text that makes the mathematics of the college algebra course accessible to all students.

Throughout the text, solutions to many examples are presented from multiple perspectives—algebraically, graphically, and numerically. The side-by-side format of this pedagogical feature helps students to see that a problem can be solved in more than one way and to see that different methods yield the same result. The side-by-side format also addresses many different learning styles.

We have found that many college algebra students grasp mathematical concepts more easily when they work with them in the context of real-life situations. Students have numerous opportunities to do this throughout this text. The *Make a Decision* feature further connects real-life data and applications and motivates students. It also offers students the opportunity to generate and analyze mathematical models from large data sets. To reinforce the concept of functions, we have compiled all the elementary functions as a *Library of Parent Functions*, presented in a summary on the endpapers of the text for convenient reference. Each function is introduced at the first point of use in the text with a definition and description of basic characteristics.

We have carefully written and designed each page to make the book more readable and accessible to students. For example, to avoid unnecessary page turning and disruptions to students' thought processes, each example and corresponding solution begins and ends on the same page.

Supports Student Success

During more than 30 years of teaching and writing, we have learned many things about the teaching and learning of mathematics. We have found that students are most successful when they know what they are expected to learn and why it is important to learn the concepts. With that in mind, we have incorporated a thematic study thread throughout this textbook.

Each chapter begins with a list of applications that are covered in the chapter and serve as a motivational tool by connecting section content to real-life situations. Using the same pedagogical theme, each section begins with a set of section learning objectives—*What You Should Learn*. These are followed by an engaging real-life application—*Why You Should Learn It*—that motivates students and illustrates an area where the mathematical concepts will be applied in an example or exercise in the section. *The Chapter Summary—What Did You Learn?*—at the end of each chapter includes *Key Terms* with page references and *Key Concepts*, organized by section, that were covered throughout the chapter. The *Chapter Summary* serves as a useful study aid for students.

Throughout the text, other features further improve accessibility. *Study Tips* are provided throughout the text at point-of-use to reinforce concepts and to help students learn how to study mathematics. *Explorations* reinforce mathematical concepts. Each example with worked-out solution is followed by a *Checkpoint*, which directs the student to work a similar exercise from the exercise set. The *Section Exercises* begin with a *Vocabulary Check*, which gives the students an opportunity to test their understanding of the important terms in the section. A *Prerequisites Skills* is offered in margin notes throughout the textbook exposition. Reviewing the prerequisite skills will enable students to master new concepts more quickly. *Synthesis Exercises* check students' conceptual understanding of the topics in each section. *Skills Review Exercises* provide additional practice with the concepts in the chapter or previous chapters. *Review Exercises*, *Chapter Tests*, and periodic *Cumulative Tests* offer students frequent opportunities for self-assessment and to develop strong study and test-taking skills. The *Progressive Summaries* and the *Study Capsules* serve as a quick reference when working on homework or as a cumulative study aid.

The use of technology also supports students with different learning styles, and graphing calculators are fully integrated into the text presentation. The *Technology Support Appendix* makes it easier for students to use technology. *Technology Support* notes are provided throughout the text at point-of-use. These notes guide students to the *Technology Support Appendix*, where they can learn how to use specific graphing calculator features to enhance their understanding of the concepts presented in the text. These notes also direct students to the *Graphing Technology Guide*, in the *Online Study Center*, for keystroke support that is available for numerous calculator models. *Technology Tips* are provided in the text at point-of-use to call attention to the strengths and weaknesses of graphing technology, as well as to offer alternative methods for solving or checking a problem using technology. Because students are often misled by the limitations of graphing calculators, we have, where appropriate, used color to enhance the graphing calculator displays in the textbook. This enables students to visualize the mathematical concepts clearly and accurately and avoid common misunderstandings.

Numerous additional text-specific resources are available to help students succeed in the college algebra course. These include "live" online tutoring, instructional DVDs, and a variety of other resources, such as tutorial support and self-assessment, which are available on the Web and in Eduspace®. In addition, the *Online Notetaking Guide* is a notetaking guide that helps students organize their class notes and create an effective study and review tool.

Flexible Options for Instructors

From the time we first began writing textbooks in the early 1970s, we have always considered it a critical part of our role as authors to provide instructors with flexible programs. In addition to addressing a variety of learning styles, the optional features within the text allow instructors to design their courses to meet their instructional needs and the needs of their students. For example, the *Explorations* throughout the text can be used as a quick introduction to concepts or as a way to reinforce student understanding.

Our goal when developing the exercise sets was to address a wide variety of learning styles and teaching preferences. The *Vocabulary Check* questions are provided at the beginning of every exercise set to help students learn proper mathematical terminology. In each exercise set we have included a variety of exercise types, including questions requiring writing and critical thinking, as well as real-data applications. The problems are carefully graded in difficulty from mastery of basic skills to more challenging exercises. Some of the more challenging exercises include the *Synthesis Exercises* that combine skills and are used to check for conceptual understanding, and the *Make a Decision* exercises that further connect real-life data and applications and motivate students. *Skills Review Exercises*, placed at the end of each exercise set, reinforce previously learned skills. The *Proofs in Mathematics*, at the end of each chapter, are proofs of important mathematical properties and theorems and illustrate various proof techniques. This feature gives the instructors the opportunity to incorporate more rigor into their course. In addition, Houghton Mifflin's Eduspace® website offers instructors the option to assign homework and tests online—and also includes the ability to grade these assignments automatically.

Several other print and media resources are available to support instructors. The *Online Instructor Success Organizer* includes suggested lesson plans and is an especially useful tool for larger departments that want all sections of a course to follow the same outline. The *Instructor's Edition* of the *Online Student Notetaking Guide* can be used as a lecture outline for every section of the text and includes additional examples for classroom discussion and important definitions. This is another valuable resource for schools trying to have consistent instruction and it can be used as a resource to support less experienced instructors. When used in conjunction with the *Online Student Notetaking Guide* these resources can save instructors preparation time and help students concentrate on important concepts.

Instructors who stress applications and problem solving and integrate technology into their course will be able to use this text successfully.

We hope you enjoy the Fifth Edition.

Ron Larson
Robert Hostetler
Bruce H. Edwards

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We would like to thank the many people who have helped us prepare the text and supplements package, including all those reviewers who have contributed to this and previous editions of the text. Their encouragement, criticisms, and suggestions have been invaluable to us.

Reviewers

Tony Homayoon Akhlaghi
Bellevue Community College

Daniel D. Anderson
University of Iowa

Bruce Armbrust
Lake Tahoe Community College

Jamie Whitehead Ashby
Texarkana College

Teresa Barton
Western New England College

Kimberly Bennekin
Georgia Perimeter College

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Humboldt State University

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Northern Virginia Community College

Spencer Hurd
The Citadel

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Missouri Western State College

Deborah Johnson
Cambridge South Dorchester High School

Francine Winston Johnson
Howard Community College

Luella Johnson
State University of New York, College at Buffalo

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Seminole Community College

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University of Akron

Laura Reger
Milwaukee Area Technical College

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Georgia Southern University

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Stephen Slack
Kenyon College

Judith Smalling
St. Petersburg Junior College

Pamela K. M. Smith
Fort Lewis College

Cathryn U. Stark
Collin County Community College

Craig M. Steenberg
Lewis-Clark State College

Mary Jane Sterling
Bradley University

G. Bryan Stewart
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Mahbobeh Vezvaei
Kent State University

Ellen Vilas
York Technical College

Hayat Weiss
Middlesex Community College

Howard L. Wilson
Oregon State University

Joel E. Wilson
Eastern Kentucky University

Michelle Wilson
Franklin University

Fred Worth
Henderson State University

Karl M. Zilm
Lewis and Clark Community College

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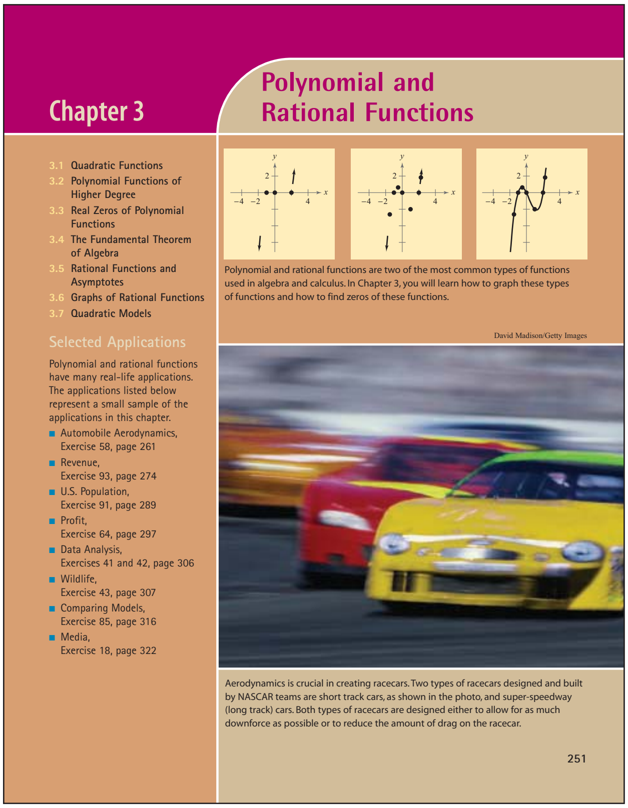
If you have suggestions for improving this text, please feel free to write us. Over the past two decades we have received many useful comments from both instructors and students, and we value these very much.

Ron Larson
Robert Hostetler
Bruce H. Edwards

Features Highlights



Chapter 3

Polynomial and Rational Functions

Selected Applications

Polynomial and rational functions have many real-life applications. The applications listed below represent a small sample of the applications in this chapter.

- Automobile Aerodynamics, Exercise 58, page 261
- Revenue, Exercise 93, page 274
- U.S. Population, Exercise 91, page 289
- Profit, Exercise 64, page 297
- Data Analysis, Exercises 41 and 42, page 306
- Wildlife, Exercise 43, page 307
- Comparing Models, Exercise 85, page 316
- Media, Exercise 18, page 322

Polynomial and rational functions are two of the most common types of functions used in algebra and calculus. In Chapter 3, you will learn how to graph these types of functions and how to find zeros of these functions.

David Madison/Getty Images

Aerodynamics is crucial in creating racecars. Two types of racecars designed and built by NASCAR teams are short track cars, as shown in the photo, and super-speedway (long track) cars. Both types of racecars are designed either to allow for as much downforce as possible or to reduce the amount of drag on the racecar.

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Chapter Opener

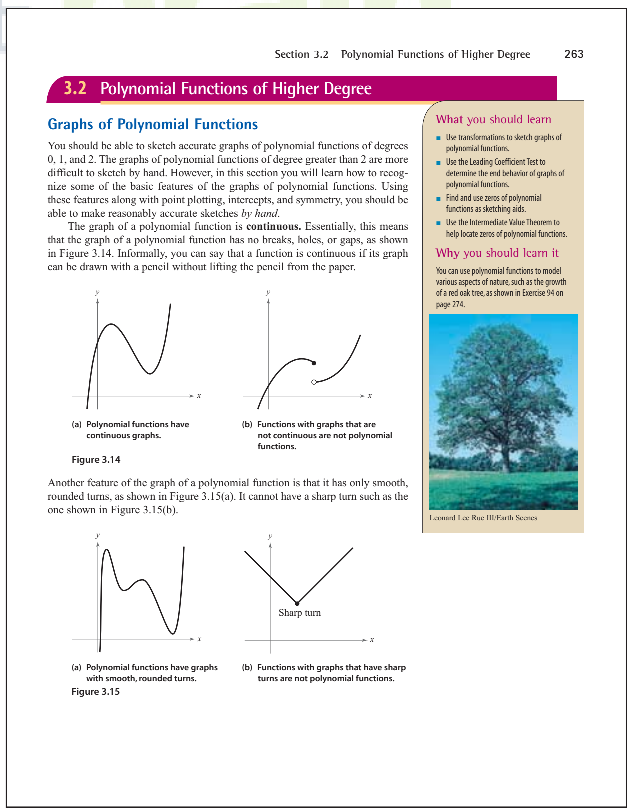
Each chapter begins with a comprehensive overview of the chapter concepts. The photograph and caption illustrate a real-life application of a key concept. Section references help students prepare for the chapter.

Applications List

An abridged list of applications, covered in the chapter, serve as a motivational tool by connecting section content to real-life situations.

"What You Should Learn" and "Why You Should Learn It"

Sections begin with *What You Should Learn*, an outline of the main concepts covered in the section, and *Why You Should Learn It*, a real-life application or mathematical reference that illustrates the relevance of the section content.



Section 3.2 Polynomial Functions of Higher Degree 263

3.2 Polynomial Functions of Higher Degree

Graphs of Polynomial Functions

You should be able to sketch accurate graphs of polynomial functions of degrees 0, 1, and 2. The graphs of polynomial functions of degree greater than 2 are more difficult to sketch by hand. However, in this section you will learn how to recognize some of the basic features of the graphs of polynomial functions. Using these features along with point plotting, intercepts, and symmetry, you should be able to make reasonably accurate sketches *by hand*.

The graph of a polynomial function is **continuous.** Essentially, this means that the graph of a polynomial function has no breaks, holes, or gaps, as shown in Figure 3.14. Informally, you can say that a function is continuous if its graph can be drawn with a pencil without lifting the pencil from the paper.

(a) Polynomial functions have continuous graphs. (b) Functions with graphs that are not continuous are not polynomial functions.

Figure 3.14

Another feature of the graph of a polynomial function is that it has only smooth, rounded turns, as shown in Figure 3.15(a). It cannot have a sharp turn such as the one shown in Figure 3.15(b).

Sharp turn

(a) Polynomial functions have graphs with smooth, rounded turns. (b) Functions with graphs that have sharp turns are not polynomial functions.

Figure 3.15

What you should learn

- Use transformations to sketch graphs of polynomial functions.
- Use the Leading Coefficient Test to determine the end behavior of graphs of polynomial functions.
- Find and use zeros of polynomial functions as sketching aids.
- Use the Intermediate Value Theorem to help locate zeros of polynomial functions.

Why you should learn it

You can use polynomial functions to model various aspects of nature, such as the growth of a red oak tree, as shown in Exercise 94 on page 274.

Leonard Lee Rue III/Earth Scenes

FEATURES

366 Chapter 4 Exponential and Logarithmic Functions

Example 4 Solving an Exponential Equation

Solve $2(3^{2t-5}) - 4 = 11$.

Solution

$2(3^{2t-5}) - 4 = 11$ Write original equation.

$2(3^{2t-5}) = 15$ Add 4 to each side.

$3^{2t-5} = \frac{15}{2}$ Divide each side by 2.

$\log_3 3^{2t-5} = \log_3 \frac{15}{2}$ Take log (base 3) of each side.

$2t - 5 = \log_3 \frac{15}{2}$ Inverse Property

$2t = 5 + \log_3 7.5$ Add 5 to each side.

$t = \frac{5}{2} + \frac{1}{2}\log_3 7.5$ Divide each side by 2.

$t \approx 3.42$ Use a calculator.

The solution is $t = \frac{5}{2} + \frac{1}{2}\log_3 7.5 \approx 3.42$. Check this in the original equation.

CHECKPOINT Now try Exercise 49.

STUDY TIP

Remember that to evaluate a logarithm such as $\log_3 7.5$, you need to use the change-of-base formula.

$$\log_3 7.5 = \frac{\ln 7.5}{\ln 3} \approx 1.834$$

When an equation involves two or more exponential expressions, you can still use a procedure similar to that demonstrated in the previous three examples. However, the algebra is a bit more complicated.

Example 5 Solving an Exponential Equation in Quadratic Form

Solve $e^{2x} - 3e^x + 2 = 0$.

Algebraic Solution

$e^{2x} - 3e^x + 2 = 0$ Write original equation.

$(e^x)^2 - 3e^x + 2 = 0$ Write in quadratic form.

$(e^x - 2)(e^x - 1) = 0$ Factor.

$e^x - 2 = 0$ Set 1st factor equal to 0.

$e^x = 2$ Add 2 to each side.

$x = \ln 2$ Solution

$e^x - 1 = 0$ Set 2nd factor equal to 0.

$e^x = 1$ Add 1 to each side.

$x = \ln 1$ Inverse Property

$x = 0$ Solution

The solutions are $x = \ln 2 \approx 0.69$ and $x = 0$. Check these in the original equation.

CHECKPOINT Now try Exercise 61.

Graphical Solution

Use a graphing utility to graph $y = e^{2x} - 3e^x + 2$. Use the *zero* or *root* feature or the *zoom* and *trace* features of the graphing utility to approximate the values of x for which $y = 0$. In Figure 4.35, you can see that the zeros occur at $x = 0$ and at $x \approx 0.69$. So, the solutions are $x = 0$ and $x \approx 0.69$.

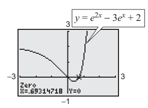


Figure 4.35

Examples

Many examples present side-by-side solutions with multiple approaches—algebraic, graphical, and numerical. This format addresses a variety of learning styles and shows students that different solution methods yield the same result.

Checkpoint

The *Checkpoint* directs students to work a similar problem in the exercise set for extra practice.

Study Tips

Study Tips reinforce concepts and help students learn how to study mathematics.

Library of Parent Functions

The *Library of Parent Functions* feature defines each elementary function and its characteristics at first point of use. The *Study Capsules* are also referenced for further review of each elementary function.

Explorations

The *Explorations* engage students in active discovery of mathematical concepts, strengthen critical thinking skills, and help them to develop an intuitive understanding of theoretical concepts.

New! Prerequisite Skills

A review of algebra skills needed to complete the examples is offered to the students at point of use throughout the text.

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Library of Parent Functions: Polynomial Function

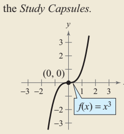
The graphs of polynomial functions of degree 1 are lines, and those of functions of degree 2 are parabolas. The graphs of all polynomial functions are smooth and continuous. A polynomial function of degree n has the form

$$f(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_2x^2 + a_1x + a_0$$

where n is a positive integer and $a_n \neq 0$. The polynomial functions that have the simplest graphs are monomials of the form $f(x) = x^n$, where n is an integer greater than zero. If n is even, the graph is similar to the graph of $f(x) = x^2$ and touches the axis at the x-intercept. If n is odd, the graph is similar to the graph of $f(x) = x^3$ and crosses the axis at the x-intercept. The greater the value of n, the flatter the graph near the origin. The basic characteristics of the *cubic function* $f(x) = x^3$ are summarized below. A review of polynomial functions can be found in the *Study Capsules*.

Graph of $f(x) = x^3$

Domain: $(-\infty, \infty)$
Range: $(-\infty, \infty)$
Intercept: $(0, 0)$
Increasing on $(-\infty, \infty)$
Odd function
Origin symmetry



Exploration

Use a graphing utility to graph $y = x^n$ for $n = 2, 4$, and 8. (Use the viewing window $-1.5 \le x \le 1.5$ and $-1 \le y \le 6$.) Compare the graphs. In the interval $(-1, 1)$, which graph is on the bottom? Outside the interval $(-1, 1)$, which graph is on the bottom?

Use a graphing utility to graph $y = x^n$ for $n = 3, 5$, and 7. (Use the viewing window $-1.5 \le x \le 1.5$ and $-4 \le y \le 4$.) Compare the graphs. In the intervals $(-\infty, -1)$ and $(0, 1)$, which graph is on the bottom? In the intervals $(-1, 0)$ and $(1, \infty)$, which graph is on the bottom?

Example 1 Transformations of Monomial Functions

Sketch the graphs of (a) $f(x) = -x^5$, (b) $g(x) = x^4 + 1$, and (c) $h(x) = (x + 1)^4$.

Solution

a. Because the degree of $f(x) = -x^5$ is odd, the graph is similar to the graph of $y = x^3$. Moreover, the negative coefficient reflects the graph in the x-axis, as shown in Figure 3.16.

b. The graph of $g(x) = x^4 + 1$ is an upward shift of one unit of the graph of $y = x^4$, as shown in Figure 3.17.

c. The graph of $h(x) = (x + 1)^4$ is a left shift of one unit of the graph of $y = x^4$, as shown in Figure 3.18.

Prerequisite Skills

If you have difficulty with this example, review shifting and reflecting of graphs in Section 1.5.

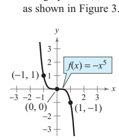


Figure 3.16

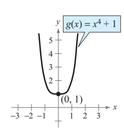


Figure 3.17

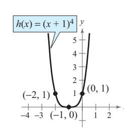
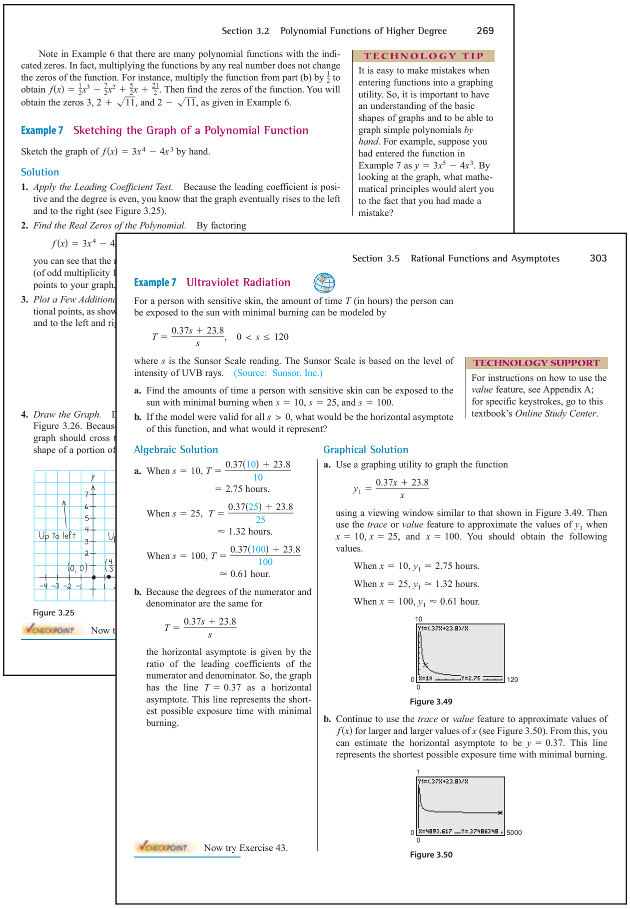


Figure 3.18

CHECKPOINT Now try Exercise 9.



Technology Tip

Technology Tips point out the pros and cons of technology use in certain mathematical situations. *Technology Tips* also provide alternative methods of solving or checking a problem by the use of a graphing calculator.

Technology Support

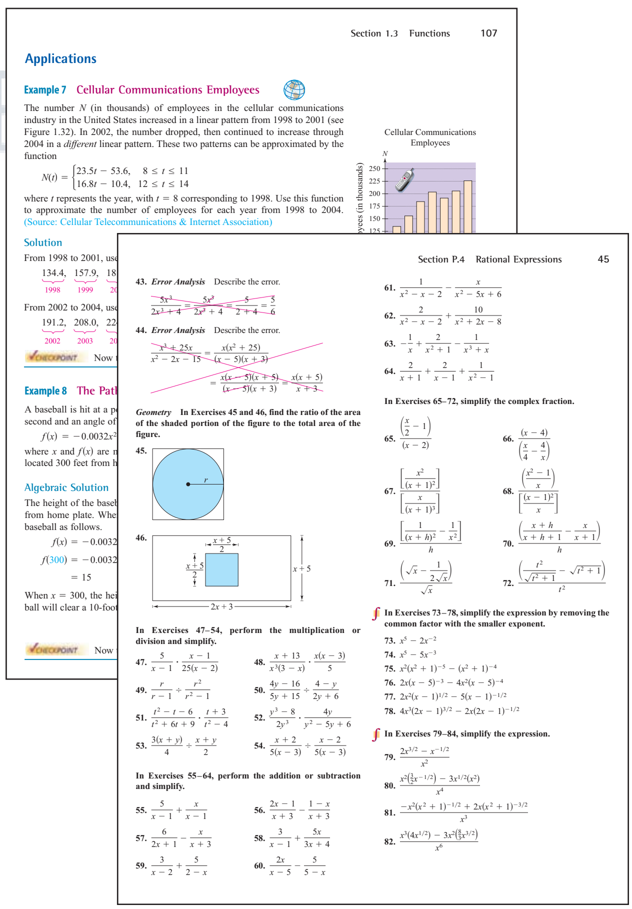
The *Technology Support* feature guides students to the *Technology Support Appendix* if they need to reference a specific calculator feature. These notes also direct students to the *Graphing Technology Guide*, in the *Online Study Center*, for keystroke support that is available for numerous calculator models.

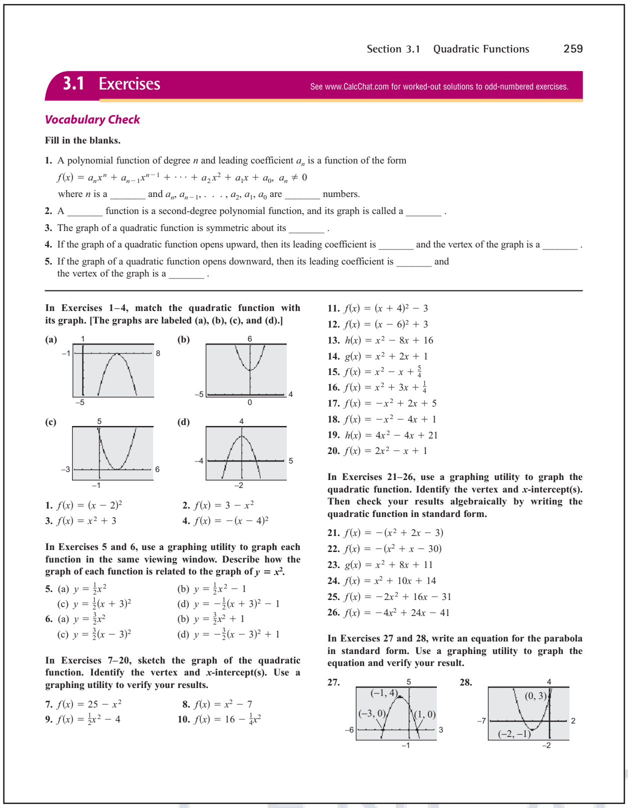
Real-Life Applications

A wide variety of real-life applications, many using current real data, are integrated throughout the examples and exercises. The indicates an example that involves a real-life application.

Algebra of Calculus

Throughout the text, special emphasis is given to the algebraic techniques used in calculus. *Algebra of Calculus* examples and exercises are integrated throughout the text and are identified by the symbol $\int$.

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3.1 Exercises See www.CalcChat.com for worked-out solutions to odd-numbered exercises.

Vocabulary Check

Fill in the blanks.

1. A polynomial function of degree n and leading coefficient a_n is a function of the form $f(x) = a_nx^n + a_{n-1}x^{n-1} + \cdots + a_2x^2 + a_1x + a_0,\ a_n \neq 0$ where n is a _______ and $a_n, a_{n-1}, \ldots, a_2, a_1, a_0$ are _______ numbers.
2. A _______ function is a second-degree polynomial function, and its graph is called a _______ .
3. The graph of a quadratic function is symmetric about its _______ .
4. If the graph of a quadratic function opens upward, then its leading coefficient is _______ and the vertex of the graph is a _______ .
5. If the graph of a quadratic function opens downward, then its leading coefficient is _______ and the vertex of the graph is a _______ .

In Exercises 1–4, match the quadratic function with its graph. [The graphs are labeled (a), (b), (c), and (d).]

(a) (b) (c) (d)

1. $f(x) = (x-2)^2$
2. $f(x) = 3 - x^2$
3. $f(x) = x^2 + 3$
4. $f(x) = -(x-4)^2$

In Exercises 5 and 6, use a graphing utility to graph each function in the same viewing window. Describe how the graph of each function is related to the graph of $y = x^2$.

5. (a) $y = \frac{1}{2}x^2$ (b) $y = \frac{1}{2}x^2 - 1$ (c) $y = \frac{1}{2}(x+3)^2$ (d) $y = -\frac{1}{2}(x+3)^2 - 1$
6. (a) $y = \frac{3}{2}x^2$ (b) $y = \frac{3}{2}x^2 + 1$ (c) $y = \frac{3}{2}(x-3)^2$ (d) $y = -\frac{3}{2}(x-3)^2 + 1$

In Exercises 7–20, sketch the graph of the quadratic function. Identify the vertex and x-intercept(s). Use a graphing utility to verify your results.

7. $f(x) = 25 - x^2$
8. $f(x) = x^2 - 7$
9. $f(x) = \frac{1}{2}x^2 - 4$
10. $f(x) = 16 - \frac{1}{4}x^2$
11. $f(x) = (x+4)^2 - 3$
12. $f(x) = (x-6)^2 + 3$
13. $h(x) = x^2 - 8x + 16$
14. $g(x) = x^2 + 2x + 1$
15. $f(x) = x^2 - x + \frac{5}{4}$
16. $f(x) = x^2 + 3x + \frac{1}{4}$
17. $f(x) = -x^2 + 2x + 5$
18. $f(x) = -x^2 - 4x + 1$
19. $h(x) = 4x^2 - 4x + 21$
20. $f(x) = 2x^2 - x + 1$

In Exercises 21–26, use a graphing utility to graph the quadratic function. Identify the vertex and x-intercept(s). Then check your results algebraically by writing the quadratic function in standard form.

21. $f(x) = -(x^2 + 2x - 3)$
22. $f(x) = -(x^2 + x - 30)$
23. $g(x) = x^2 + 8x + 11$
24. $f(x) = x^2 + 10x + 14$
25. $f(x) = -2x^2 + 16x - 31$
26. $f(x) = -4x^2 + 24x - 41$

In Exercises 27 and 28, write an equation for the parabola in standard form. Use a graphing utility to graph the equation and verify your result.

27. (−1, 4), (−3, 0), (1, 0)
28. (0, 3), (−2, −1)

Section Exercises

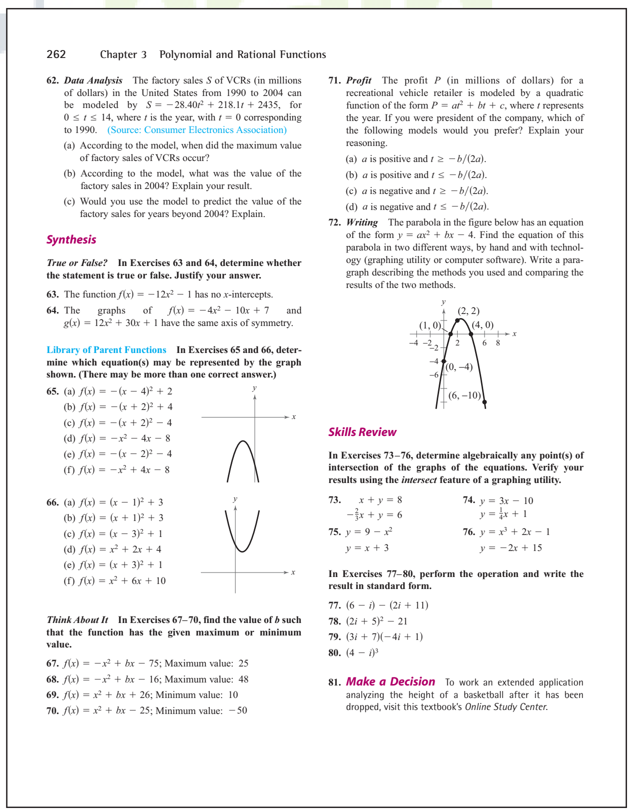
The section exercise sets consist of a variety of computational, conceptual, and applied problems.

Vocabulary Check

Section exercises begin with a *Vocabulary Check* that serves as a review of the important mathematical terms in each section.

New! Calc Chat

The worked-out solutions to the odd-numbered text exercises are now available at *www.CalcChat.com*.


262 Chapter 3 Polynomial and Rational Functions

62. ***Data Analysis*** The factory sales S of VCRs (in millions of dollars) in the United States from 1990 to 2004 can be modeled by $S = -28.40t^2 + 218.1t + 2435$, for $0 \le t \le 14$, where t is the year, with $t = 0$ corresponding to 1990. (Source: Consumer Electronics Association)
 (a) According to the model, when did the maximum value of factory sales of VCRs occur?
 (b) According to the model, what was the value of the factory sales in 2004? Explain your result.
 (c) Would you use the model to predict the value of the factory sales for years beyond 2004? Explain.

Synthesis

True or False? **In Exercises 63 and 64, determine whether the statement is true or false. Justify your answer.**

63. The function $f(x) = -12x^2 - 1$ has no x-intercepts.
64. The graphs of $f(x) = -4x^2 - 10x + 7$ and $g(x) = 12x^2 + 30x + 1$ have the same axis of symmetry.

Library of Parent Functions **In Exercises 65 and 66, determine which equation(s) may be represented by the graph shown. (There may be more than one correct answer.)**

65. (a) $f(x) = -(x-4)^2 + 2$ (b) $f(x) = -(x+2)^2 + 4$ (c) $f(x) = -(x+2)^2 - 4$ (d) $f(x) = -x^2 - 4x - 8$ (e) $f(x) = -(x-2)^2 - 4$ (f) $f(x) = -x^2 + 4x - 8$
66. (a) $f(x) = (x-1)^2 + 3$ (b) $f(x) = (x+1)^2 + 3$ (c) $f(x) = (x-3)^2 + 1$ (d) $f(x) = x^2 + 2x + 4$ (e) $f(x) = (x+3)^2 + 1$ (f) $f(x) = x^2 + 6x + 10$

Think About It **In Exercises 67–70, find the value of b such that the function has the given maximum or minimum value.**

67. $f(x) = -x^2 + bx - 75$; Maximum value: 25
68. $f(x) = -x^2 + bx - 16$; Maximum value: 48
69. $f(x) = x^2 + bx + 26$; Minimum value: 10
70. $f(x) = x^2 + bx - 25$; Minimum value: -50
71. ***Profit*** The profit P (in millions of dollars) for a recreational vehicle retailer is modeled by a quadratic function of the form $P = at^2 + bt + c$, where t represents the year. If you were president of the company, which of the following models would you prefer? Explain your reasoning.
 (a) a is positive and $t \ge -b/(2a)$.
 (b) a is positive and $t \le -b/(2a)$.
 (c) a is negative and $t \ge -b/(2a)$.
 (d) a is negative and $t \le -b/(2a)$.
72. ***Writing*** The parabola in the figure below has an equation of the form $y = ax^2 + bx - 4$. Find the equation of this parabola in two different ways, by hand and with technology (graphing utility or computer software). Write a paragraph describing the methods you used and comparing the results of the two methods.

(1, 0), (2, 2), (4, 0), (0, −4), (6, −10)

Skills Review

In Exercises 73–76, determine algebraically any point(s) of intersection of the graphs of the equations. Verify your results using the *intersect* feature of a graphing utility.

73. $x + y = 8$, $-\frac{2}{3}x + y = 6$
74. $y = 3x - 10$, $y = \frac{1}{4}x + 1$
75. $y = 9 - x^2$, $y = x + 3$
76. $y = x^3 + 2x - 1$, $y = -2x + 15$

In Exercises 77–80, perform the operation and write the result in standard form.

77. $(6 - i) - (2i + 11)$
78. $(2i + 5)^2 - 21$
79. $(3i + 7)(-4i + 1)$
80. $(4 - i)^3$
81. ***Make a Decision*** To work an extended application analyzing the height of a basketball after it has been dropped, visit this textbook's *Online Study Center*.

Synthesis and Skills Review Exercises

Each exercise set concludes with three types of exercises.

Synthesis exercises promote further exploration of mathematical concepts, critical thinking skills, and writing about mathematics. The exercises require students to show their understanding of the relationships between many concepts in the section.

Skills Review Exercises reinforce previously learned skills and concepts.

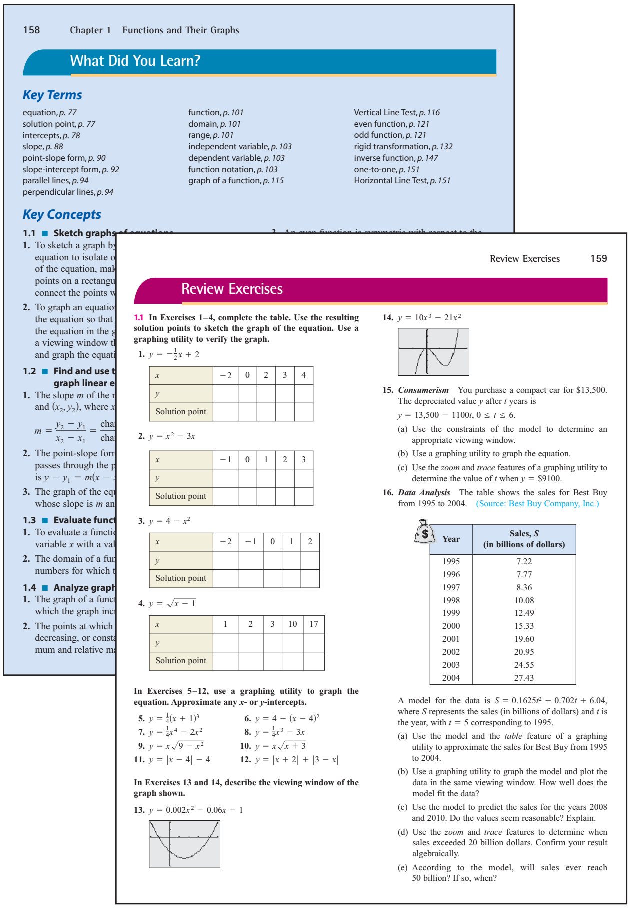
New! *Make a Decision* exercises, found in selected sections, further connect real-life data and applications and motivate students. They also offer students the opportunity to generate and analyze mathematical models from large data sets.

Chapter Summary

The *Chapter Summary* "*What Did You Learn*?" includes *Key Terms* with page references and *Key Concepts*, organized by section, that were covered throughout the chapter.

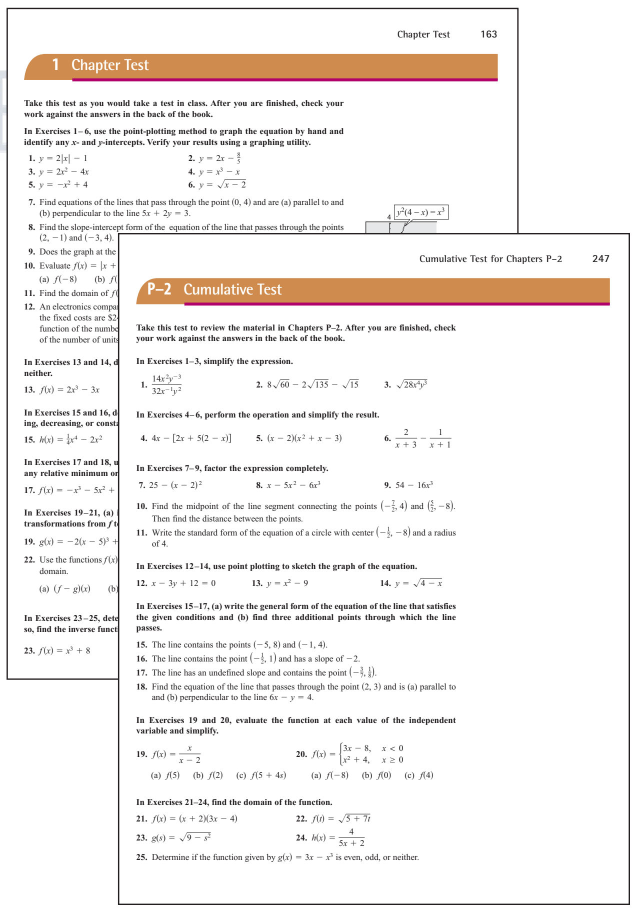
Review Exercises

The chapter *Review Exercises* provide additional practice with the concepts covered in the chapter.

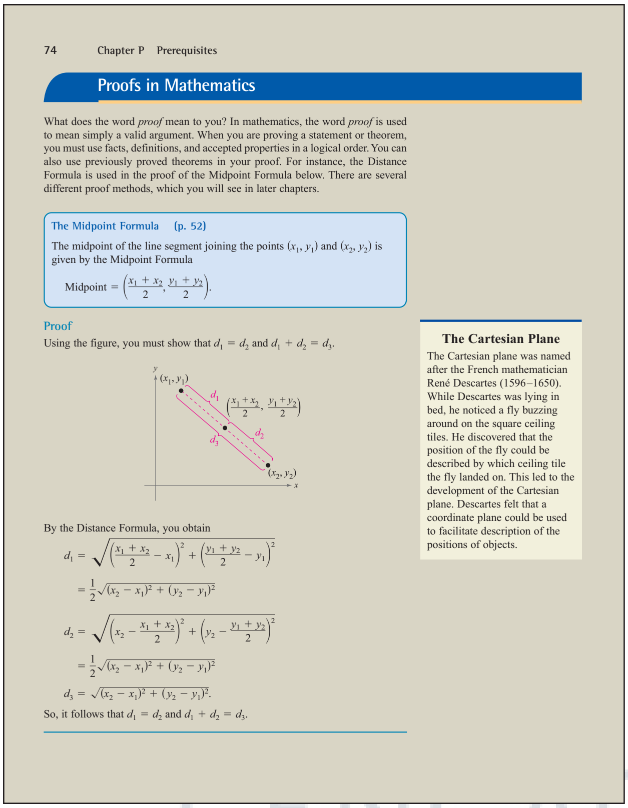


Chapter Tests and Cumulative Tests

Chapter Tests, at the end of each chapter, and periodic *Cumulative Tests* offer students frequent opportunities for self-assessment and to develop strong study and test-taking skills.



FEATURES



74 Chapter P Prerequisites

Proofs in Mathematics

What does the word *proof* mean to you? In mathematics, the word *proof* is used to mean simply a valid argument. When you are proving a statement or theorem, you must use facts, definitions, and accepted properties in a logical order. You can also use previously proved theorems in your proof. For instance, the Distance Formula is used in the proof of the Midpoint Formula below. There are several different proof methods, which you will see in later chapters.

The Midpoint Formula (p. 52)

The midpoint of the line segment joining the points (x_1, y_1) and (x_2, y_2) is given by the Midpoint Formula

$$\text{Midpoint} = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right).$$

Proof

Using the figure, you must show that $d_1 = d_2$ and $d_1 + d_2 = d_3$.

By the Distance Formula, you obtain

$$d_1 = \sqrt{\left(\frac{x_1 + x_2}{2} - x_1\right)^2 + \left(\frac{y_1 + y_2}{2} - y_1\right)^2}$$
$$= \frac{1}{2}\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$d_2 = \sqrt{\left(x_2 - \frac{x_1 + x_2}{2}\right)^2 + \left(y_2 - \frac{y_1 + y_2}{2}\right)^2}$$
$$= \frac{1}{2}\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$d_3 = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}.$$

So, it follows that $d_1 = d_2$ and $d_1 + d_2 = d_3$.

The Cartesian Plane

The Cartesian plane was named after the French mathematician René Descartes (1596–1650). While Descartes was lying in bed, he noticed a fly buzzing around on the square ceiling tiles. He discovered that the position of the fly could be described by which ceiling tile the fly landed on. This led to the development of the Cartesian plane. Descartes felt that a coordinate plane could be used to facilitate description of the positions of objects.

Proofs in Mathematics

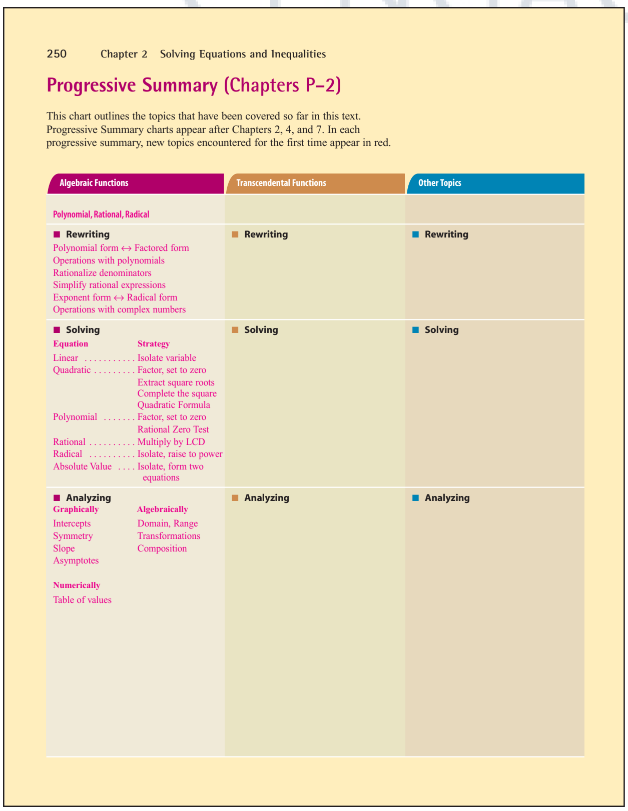
At the end of every chapter, proofs of important mathematical properties and theorems are presented as well as discussions of various proof techniques.

New! Progressive Summaries

The *Progressive Summaries* are a series of charts that are usually placed at the end of every third chapter. Each *Progressive Summary* is completed in a gradual manner as new concepts are covered. Students can use the *Progressive Summaries* as a cumulative study aid and to see the connection between concepts and skills.

New! Study Capsules

Each *Study Capsule* in Appendix F summarizes many of the key concepts covered in previous chapters. A *Study Capsule* provides definitions, examples, and procedures for solving, simplifying, and graphing functions. Students can use this appendix as a quick reference when working on homework or studying for a test.

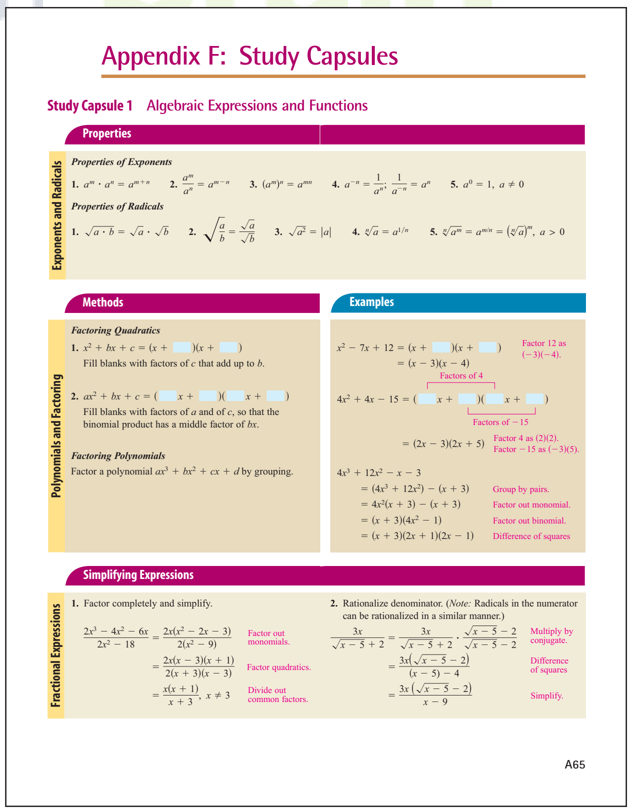


250 Chapter 2 Solving Equations and Inequalities

Progressive Summary (Chapters P–2)

This chart outlines the topics that have been covered so far in this text. Progressive Summary charts appear after Chapters 2, 4, and 7. In each progressive summary, new topics encountered for the first time appear in red.

Algebraic Functions	Transcendental Functions	Other Topics
Polynomial, Rational, Radical		
Rewriting Polynomial form ↔ Factored form Operations with polynomials Rationalize denominators Simplify rational expressions Exponent form ↔ Radical form Operations with complex numbers	**Rewriting**	**Rewriting**
Solving Equation — Strategy Linear — Isolate variable Quadratic — Factor, set to zero; Extract square roots; Complete the square; Quadratic Formula Polynomial — Factor, set to zero; Rational Zero Test Rational — Multiply by LCD Radical — Isolate, raise to power Absolute Value — Isolate, form two equations	**Solving**	**Solving**
Analyzing Graphically: Intercepts, Symmetry, Slope, Asymptotes Algebraically: Domain, Range, Transformations, Composition Numerically: Table of values	**Analyzing**	**Analyzing**



Appendix F: Study Capsules

Study Capsule 1 Algebraic Expressions and Functions

Properties — Exponents and Radicals

Properties of Exponents

1. $a^m \cdot a^n = a^{m+n}$ 2. $\frac{a^m}{a^n} = a^{m-n}$ 3. $(a^m)^n = a^{mn}$ 4. $a^{-n} = \frac{1}{a^n}; \frac{1}{a^{-n}} = a^n$ 5. $a^0 = 1, a \neq 0$

Properties of Radicals

1. $\sqrt{a \cdot b} = \sqrt{a} \cdot \sqrt{b}$ 2. $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$ 3. $\sqrt{a^2} = |a|$ 4. $\sqrt[n]{a} = a^{1/n}$ 5. $\sqrt[n]{a^m} = a^{m/n} = (\sqrt[n]{a})^m, a > 0$

Methods / Examples — Polynomials and Factoring

Factoring Quadratics

1. $x^2 + bx + c = (x + \square)(x + \square)$
Fill blanks with factors of c that add up to b.

$x^2 - 7x + 12 = (x + \square)(x + \square)$ Factor 12 as $(-3)(-4)$.
$= (x - 3)(x - 4)$ Factors of 4

2. $ax^2 + bx + c = (\square x + \square)(\square x + \square)$
Fill blanks with factors of a and of c, so that the binomial product has a middle factor of bx.

$4x^2 + 4x - 15 = (\square x + \square)(\square x + \square)$ Factors of −15
$= (2x - 3)(2x + 5)$ Factor 4 as (2)(2). Factor −15 as (−3)(5).

Factoring Polynomials

Factor a polynomial $ax^3 + bx^2 + cx + d$ by grouping.

$4x^3 + 12x^2 - x - 3$
$= (4x^3 + 12x^2) - (x + 3)$ Group by pairs.
$= 4x^2(x + 3) - (x + 3)$ Factor out monomial.
$= (x + 3)(4x^2 - 1)$ Factor out binomial.
$= (x + 3)(2x + 1)(2x - 1)$ Difference of squares

Simplifying Expressions — Fractional Expressions

1. Factor completely and simplify.

$\frac{2x^3 - 4x^2 - 6x}{2x^2 - 18} = \frac{2x(x^2 - 2x - 3)}{2(x^2 - 9)}$ Factor out monomials.
$= \frac{2x(x - 3)(x + 1)}{2(x + 3)(x - 3)}$ Factor quadratics.
$= \frac{x(x + 1)}{x + 3}, x \neq 3$ Divide out common factors.

2. Rationalize denominator. (*Note:* Radicals in the numerator can be rationalized in a similar manner.)

$\frac{3x}{\sqrt{x - 5} + 2} = \frac{3x}{\sqrt{x - 5} + 2} \cdot \frac{\sqrt{x - 5} - 2}{\sqrt{x - 5} - 2}$ Multiply by conjugate.
$= \frac{3x(\sqrt{x - 5} - 2)}{(x - 5) - 4}$ Difference of squares
$= \frac{3x(\sqrt{x - 5} - 2)}{x - 9}$ Simplify.

A65

Additional Resources—Get the Most Out of Your Textbook!

Supplements for the Instructor

Instructor's Annotated Edition (IAE)

Online Complete Solutions Guide

Online Instructor Success Organizer

Online Teaching Center

This free companion website contains an abundance of instructors resources. Visit **college.hmco.com/pic/larsonCAAGA5e** and click on the Online Teaching Center icon.

HM Testing™ (Powered by Diploma™) "Testing the way you want it"

HM Testing provides instructors all the tools they need to **create, author/edit, customize,** and **deliver** multiple types of tests. Instructors can use existing test bank content, edit the content, and author new static or algorithmic questions—all within Diploma's powerful electronic platform.

Supplements for the Student

Study and Solutions Guide

Written by the author, this manual offers step-by-step solutions for all odd-numbered text exercises as well as Chapter and Cumulative Tests. The manual also provides practice tests that are accompanied by a solution key. In addition, these worked-out solutions are available at *www.CalcChat.com*.

Online Study Center

This free companion website contains an abundance of student resources including the *Online Student Notetaking Guide*. Visit the website **college.hmco.com/pic/larsonCAAGA5e** and click on the Online Study Center icon.

Instructional DVDs

Hosted by Dana Mosely, these text-specific DVDs cover all sections of the text and provide key explanations of key concepts, examples, exercises, and applications in a lecture-based format. New to this edition, the DVDs will now include Captioning for the Hearing Impaired.

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Appendix A: Technology Support

Introduction

Graphing utilities such as graphing calculators and computers with graphing software are very valuable tools for visualizing mathematical principles, verifying solutions to equations, exploring mathematical ideas, and developing mathematical models. Although graphing utilities are extremely helpful in learning mathematics, their use does not mean that learning algebra is any less important. In fact, the combination of knowledge of mathematics and the use of graphing utilities enables you to explore mathematics more easily and to a greater depth. If you are using a graphing utility in this course, it is up to you to learn its capabilities and to practice using this tool to enhance your mathematical learning.

In this text, there are many opportunities to use a graphing utility, some of which are described below.

Uses of a Graphing Utility

1. Check or validate answers to problems obtained using algebraic methods.
2. Discover and explore algebraic properties, rules, and concepts.
3. Graph functions, and approximate solutions to equations involving functions.
4. Efficiently perform complicated mathematical procedures such as those found in many real-life applications.
5. Find mathematical models for sets of data.

In this appendix, the features of graphing utilities are discussed from a generic perspective and are listed in alphabetical order. To learn how to use the features of a specific graphing utility, consult your user's manual or go to this textbook's *Online Study Center*. Additional keystroke guides are available for most graphing utilities, and your college library may have a videotape on how to use your graphing utility.

Many graphing utilities are designed to act as "function graphers." In this course, functions and their graphs are studied in detail. You may recall from previous courses that a function can be thought of as a rule that describes the relationship between two variables. These rules are frequently written in terms of x and y. For example, the equation

$$y = 3x + 5$$

represents y as a function of x.

Many graphing utilities have an *equation editor* feature that requires that an equation be written in "$y =$" form in order to be entered, as shown in Figure A.1. (You should note that your *equation editor* screen may not look like the screen shown in Figure A.1.)

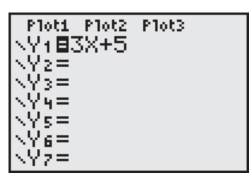

Figure A.1

Cumulative Sum Feature

The *cumulative sum* feature finds partial sums of a series. For example, to find the first four partial sums of the series

$$\sum_{k=1}^{4} 2(0.1)^k$$

choose the *cumulative sum* feature, which is found in the *operations* menu of the *list* feature (see Figure A.2). To use this feature, you will also have to use the *sequence* feature (see Figure A.2 and page A15). You must enter an expression for the sequence, a variable, the lower limit of summation, and the upper limit of summation, as shown in Figure A.3. After pressing ENTER, you can see that the first four partial sums are 0.2, 0.22, 0.222, and 0.2222. You may have to scroll to the right in order to see all the partial sums.

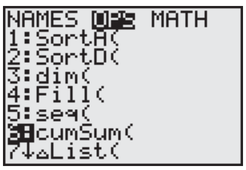

Figure A.2

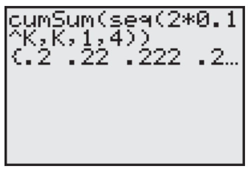

Figure A.3

TECHNOLOGY TIP

As you use your graphing utility, be aware of how parentheses are inserted in an expression. Some graphing utilities automatically insert the left parenthesis when certain calculator buttons are pressed. The placement of parentheses can make a difference between a correct answer and an incorrect answer.

Determinant Feature

The *determinant* feature evaluates the determinant of a square matrix. For example, to evaluate the determinant of the matrix shown at the right, enter the 3×3 matrix in the graphing utility using the *matrix editor*, as shown in Figure A.4. Then choose the *determinant* feature from the *math* menu of the *matrix* feature, as shown in Figure A.5. Once you choose the matrix name, A, press ENTER and you should obtain a determinant of -50, as shown in Figure A.6.

$$A = \begin{bmatrix} 7 & -1 & 0 \\ 2 & 2 & 3 \\ -6 & 4 & 1 \end{bmatrix}$$

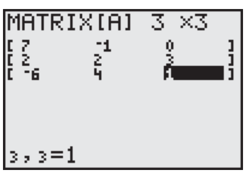

Figure A.4

Figure A.5

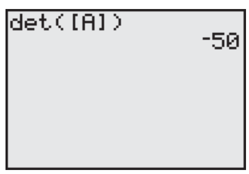

Figure A.6

Draw Inverse Feature

The *draw inverse* feature graphs the inverse function of a one-to-one function. For instance, to graph the inverse function of $f(x) = x^3 + 4$, first enter the function in the *equation editor* (see Figure A.7) and graph the function (using a square viewing window), as shown in Figure A.8. Then choose the *draw inverse* feature from the *draw* feature menu, as shown in Figure A.9. You must enter the function you want to graph the inverse function of, as shown in Figure A.10. Finally, press ENTER to obtain the graph of the inverse function of $f(x) = x^3 + 4$, as shown in Figure A.11. This feature can be used only when the graphing utility is in *function* mode.

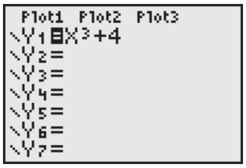

Figure A.7

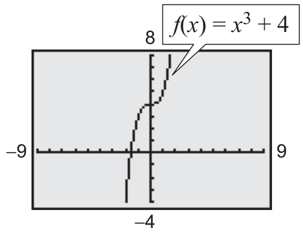

Figure A.8

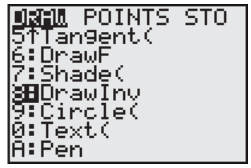

Figure A.9

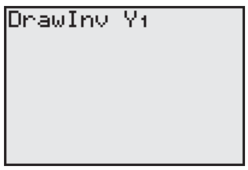

Figure A.10

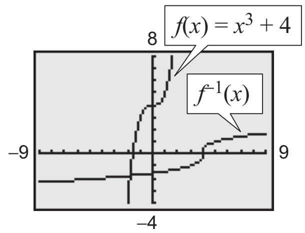

Figure A.11

Elementary Row Operations Features

Most graphing utilities can perform elementary row operations on matrices.

Row Swap Feature

The *row swap* feature interchanges two rows of a matrix. To interchange rows 1 and 3 of the matrix shown at the right, first enter the matrix in the graphing utility using the *matrix editor*, as shown in Figure A.12. Then choose the *row swap* feature from the *math* menu of the *matrix* feature, as shown in Figure A.13. When using this feature, you must enter the name of the matrix and the two rows that are to be interchanged. After pressing ENTER, you should obtain the matrix shown in Figure A.14. Because the resulting matrix will be used to demonstrate the other elementary row operation features, use the *store* feature to copy the resulting matrix to [A], as shown in Figure A.15.

$$A = \begin{bmatrix} -1 & -2 & 1 & 2 \\ 2 & -4 & 6 & -2 \\ 1 & 3 & -3 & 0 \end{bmatrix}$$

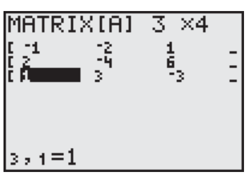

Figure A.12

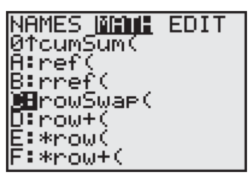

Figure A.13

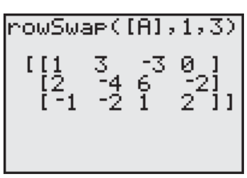

Figure A.14

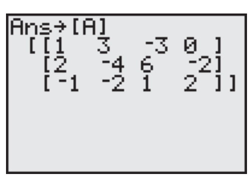

Figure A.15

TECHNOLOGY TIP

The *store* feature of a graphing utility is used to store a value in a variable or to copy one matrix to another matrix. For instance, as shown at the left, after performing a row operation on a matrix, you can copy the answer to another matrix (see Figure A.15). You can then perform another row operation on the copied matrix. If you want to continue performing row operations to obtain a matrix in row-echelon form or reduced row-echelon form, you must copy the resulting matrix to a new matrix before each operation.

Row Addition and Row Multiplication and Addition Features

The *row addition* and *row multiplication and addition* features add a row or a multiple of a row of a matrix to another row of the same matrix. To add row 1 to row 3 of the matrix stored in [A], choose the *row addition* feature from the *math* menu of the *matrix* feature, as shown in Figure A.16. When using this feature, you must enter the name of the matrix and the two rows that are to be added. After pressing ENTER, you should obtain the matrix shown in Figure A.17. Copy the resulting matrix to [A].

Figure A.16

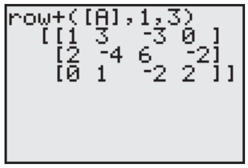

Figure A.17

To add −2 times row 1 to row 2 of the matrix stored in [A], choose the *row multiplication and addition* feature from the *math* menu of the *matrix* feature, as shown in Figure A.18. When using this feature, you must enter the constant, the name of the matrix, the row the constant is multiplied by, and the row to be added to. After pressing ENTER, you should obtain the matrix shown in Figure A.19. Copy the resulting matrix to [A].

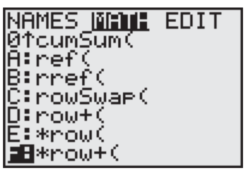

Figure A.18

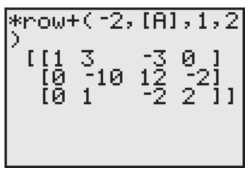

Figure A.19

Row Multiplication Feature

The *row multiplication* feature multiplies a row of a matrix by a nonzero constant. To multiply row 2 of the matrix stored in [A] by $-\frac{1}{10}$, choose the *row multiplication* feature from the *math* menu of the *matrix* feature, as shown in Figure A.20. When using this feature, you must enter the constant, the name of the matrix, and the row to be multiplied. After pressing ENTER, you should obtain the matrix shown in Figure A.21.

Figure A.20

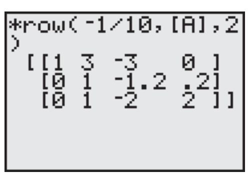

Figure A.21

Intersect Feature

The *intersect* feature finds the point(s) of intersection of two graphs. The *intersect* feature is found in the *calculate* menu (see Figure A.22). To find the point(s) of intersection of the graphs of $y_1 = -x + 2$ and $y_2 = x + 4$, first enter the equations in the *equation editor*, as shown in Figure A.23. Then graph the equations, as shown in Figure A.24. Next, use the *intersect* feature to find the point of intersection. Trace the cursor along the graph of y_1 near the intersection and press ENTER (see Figure A.25). Then trace the cursor along the graph of y_2 near the intersection and press ENTER (see Figure A.26). Marks are then placed on the graph at these points (see Figure A.27). Finally, move the cursor near the point of intersection and press ENTER. In Figure A.28, you can see that the coordinates of the point of intersection are displayed at the bottom of the window. So, the point of intersection is $(-1, 3)$.

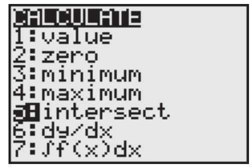

Figure A.22

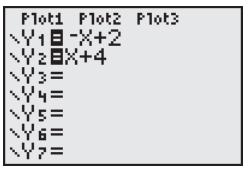

Figure A.23

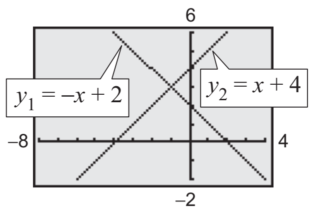

Figure A.24

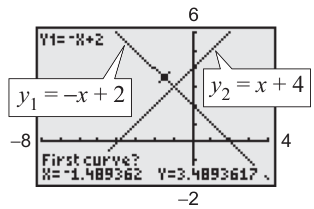

Figure A.25

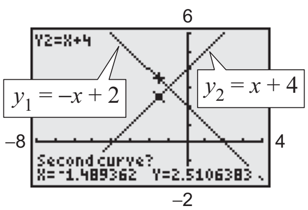

Figure A.26

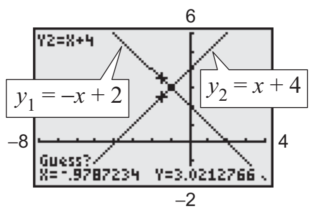

Figure A.27

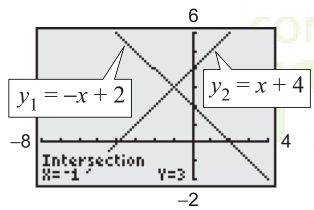

Figure A.28

List Editor

Most graphing utilities can store data in lists. The *list editor* can be used to create tables and to store statistical data. The *list editor* can be found in the *edit* menu of the *statistics* feature, as shown in Figure A.29. To enter the numbers 1 through 10 in a list, first choose a list (L_1) and then begin entering the data into each row, as shown in Figure A.30.

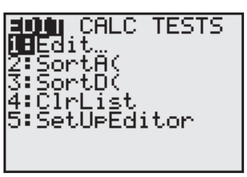

Figure A.29

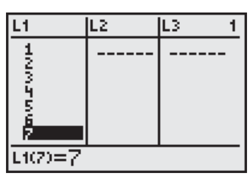

Figure A.30

You can also attach a formula to a list. For instance, you can multiply each of the data values in L_1 by 3. First, display the *list editor* and move the cursor to the top line. Then move the cursor onto the list to which you want to attach the formula (L_2). Finally, enter the formula 3* L_1 (see Figure A.31) and then press ENTER. You should obtain the list shown in Figure A.32.

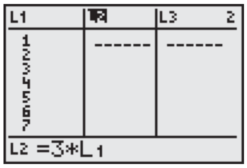

Figure A.31

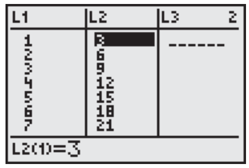

Figure A.32

Matrix Feature

The *matrix* feature of a graphing utility has many uses, such as evaluating a determinant and performing row operations.

Matrix Editor

You can define, display, and edit matrices using the *matrix editor*. The *matrix editor* can be found in the *edit* menu of the *matrix* feature. For instance, to enter the matrix shown at the right, first choose the matrix name [A], as shown in Figure A.33. Then enter the dimension of the matrix (in this case, the dimension is 2×3) and enter the entries of the matrix, as shown in Figure A.34. To display the matrix on the home screen, choose the *name* menu of the *matrix* feature and select the matrix [A] (see Figure A.35), then press ENTER. The matrix A should now appear on the home screen, as shown in Figure A.36.

$$A = \begin{bmatrix} 6 & -3 & 4 \\ 9 & 0 & -1 \end{bmatrix}$$

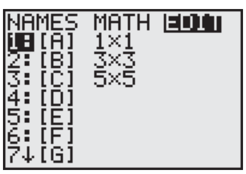

Figure A.33

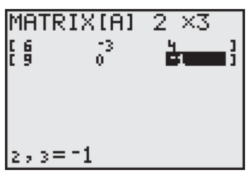

Figure A.34

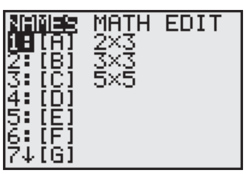

Figure A.35

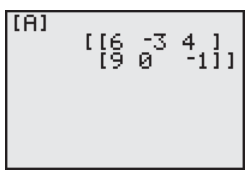

Figure A.36

Matrix Operations

Most graphing utilities can perform matrix operations. To find the sum $A + B$ of the matrices shown at the right, first enter the matrices in the *matrix editor* as [A] and [B]. Then find the sum, as shown in Figure A.37. Scalar multiplication can be performed in a similar manner. For example, you can evaluate $7A$, where A is the matrix at the right, as shown in Figure A.38. To find the product AB of the matrices A and B at the right, first be sure that the product is defined. Because the number of columns of A (2 columns) equals the number of rows of B (2 rows), you can find the product AB, as shown in Figure A.39.

$$A = \begin{bmatrix} -3 & 5 \\ 0 & 4 \end{bmatrix}$$

$$B = \begin{bmatrix} 7 & -2 \\ -1 & 2 \end{bmatrix}$$

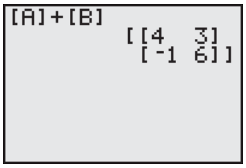

Figure A.37

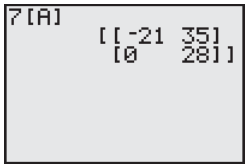

Figure A.38

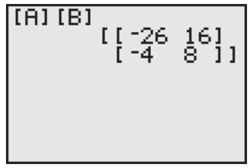

Figure A.39

Inverse Matrix

Some graphing utilities may not have an *inverse matrix* feature. However, you can find the inverse of a square matrix by using the inverse key $\boxed{x^{-1}}$. To find the inverse of the matrix shown at the right, enter the matrix in the *matrix editor* as [A]. Then find the inverse, as shown in Figure A.40.

$$A = \begin{bmatrix} 1 & -2 & 1 \\ -1 & 3 & 0 \\ 2 & 4 & 5 \end{bmatrix}$$

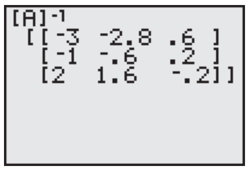

Figure A.40

Maximum and Minimum Features

The *maximum* and *minimum* features find relative extrema of a function. For instance, the graph of $y = x^3 - 3x$ is shown in Figure A.41. In the figure, the graph appears to have a relative maximum at $x = -1$ and a relative minimum at $x = 1$. To find the exact values of the relative extrema, you can use the *maximum* and *minimum* features found in the *calculate* menu (see Figure A.42). First, to find the relative maximum, choose the *maximum* feature and trace the cursor along the graph to a point left of the maximum and press ENTER (see Figure A.43). Then trace the cursor along the graph to a point right of the maximum and press ENTER (see Figure A.44). Note the two arrows near the top of the display marking the left and right bounds, as shown in Figure A.45. Next, trace the cursor along the graph between the two bounds and as close to the maximum as you can (see Figure A.45) and press ENTER. In Figure A.46, you can see that the coordinates of the maximum point are displayed at the bottom of the window. So, the relative maximum is $(-1, 2)$.

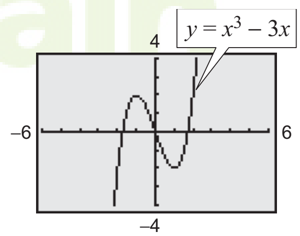

Figure A.41

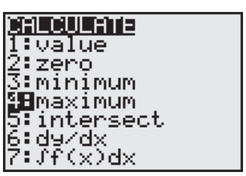

Figure A.42

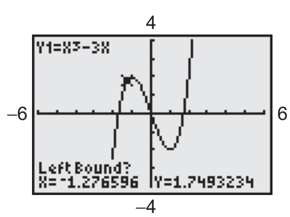

Figure A.43

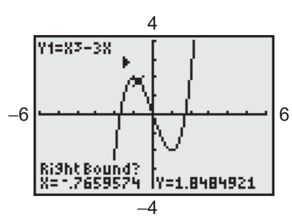

Figure A.44

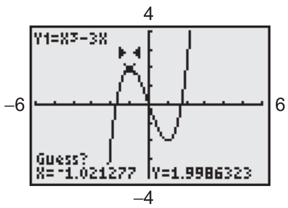

Figure A.45

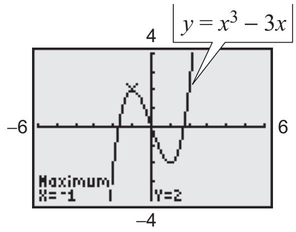

Figure A.46

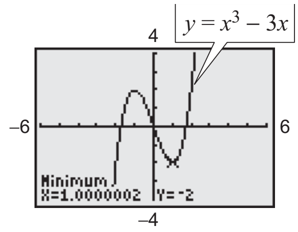

Figure A.47

You can find the relative minimum in a similar manner. In Figure A.47, you can see that the relative minimum is $(1, -2)$.

Mean and Median Features

In real-life applications, you often encounter large data sets and want to calculate statistical values. The *mean* and *median* features calculate the mean and median of a data set. For instance, in a survey, 100 people were asked how much money (in dollars) per week they withdraw from an automatic teller machine (ATM). The results are shown in the table below. The frequency represents the number of responses.

Amount	10	20	30	40	50	60	70	80	90	100
Frequency	3	8	10	19	24	13	13	7	2	1

To find the mean and median of the data set, first enter the data in the *list editor*, as shown in Figure A.48. Enter the amount in L_1 and the frequency in L_2. Then choose the *mean* feature from the *math* menu of the *list* feature, as shown in Figure A.49. When using this feature, you must enter a list and a frequency list (if applicable). In this case, the list is L_1 and the frequency list is L_2. After pressing ENTER, you should obtain a mean of \$49.80, as shown in Figure A.50. You can follow the same steps (except choose the *median* feature) to find the median of the data. You should obtain a median of \$50, as shown in Figure A.51.

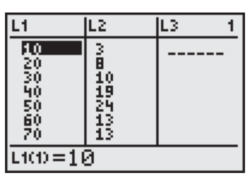

Figure A.48

Figure A.49

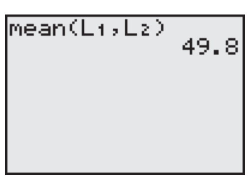

Figure A.50

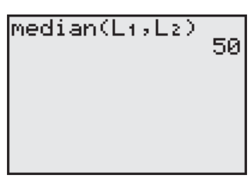

Figure A.51

Mode Settings

Mode settings of a graphing utility control how the utility displays and interprets numbers and graphs. The default mode settings are shown in Figure A.52.

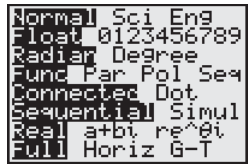

Figure A.52

Radian and Degree Modes

The trigonometric functions can be applied to angles measured in either radians or degrees. When your graphing utility is in *radian* mode, it interprets angle values as radians and displays answers in radians. When your graphing utility is in *degree* mode, it interprets angle values as degrees and displays answers in degrees. For instance, to calculate $\sin(\pi/6)$, make sure the calculator is in *radian* mode. You should obtain an answer of 0.5, as shown in Figure A.53. To calculate $\sin 45°$, make sure the calculator is in *degree* mode, as shown in Figure A.54. You should obtain an approximate answer of 0.7071, as shown in Figure A.55. If you did not change the mode of the calculator before evaluating $\sin 45°$, you would obtain an answer of approximately 0.8509, which is the sine of 45 radians.

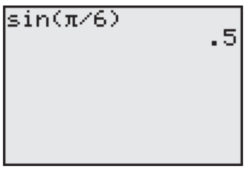

Figure A.53

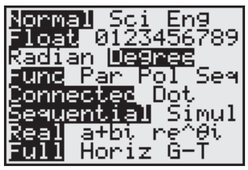

Figure A.54

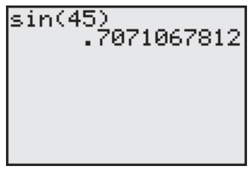

Figure A.55

Function, Parametric, Polar, and Sequence Modes

Most graphing utilities can graph using four different modes.

Function Mode The *function* mode is used to graph standard algebraic and trigonometric functions. For instance, to graph $y = 2x^2$, use the *function* mode, as shown in Figure A.52. Then enter the equation in the *equation editor*, as shown in Figure A.56. Using a standard viewing window (see Figure A.57), you obtain the graph shown in Figure A.58.

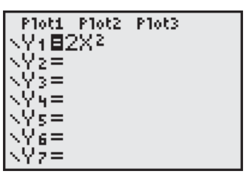

Figure A.56

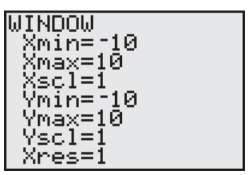

Figure A.57

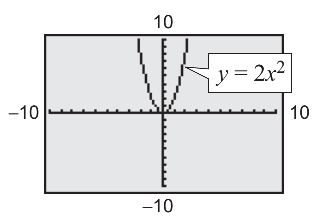

Figure A.58

Parametric Mode To graph parametric equations such as $x = t + 1$ and $y = t^2$, use the *parametric* mode, as shown in Figure A.59. Then enter the equations in the *equation editor*, as shown in Figure A.60. Using the viewing window shown in Figure A.61, you obtain the graph shown in Figure A.62.

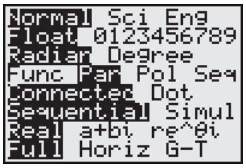

Figure A.59

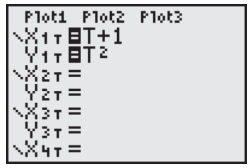

Figure A.60

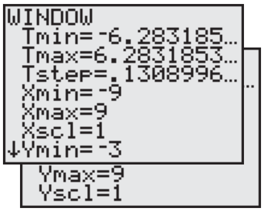

Figure A.61

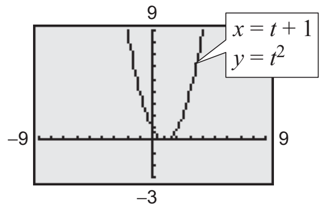

Figure A.62

Polar Mode To graph polar equations of the form $r = f(\theta)$, you can use the *polar* mode of a graphing utility. For instance, to graph the polar equation $r = 2 \cos \theta$, use the *polar* mode (and *radian* mode), as shown in Figure A.63. Then enter the equation in the *equation editor*, as shown in Figure A.64. Using the viewing window shown in Figure A.65, you obtain the graph shown in Figure A.66.

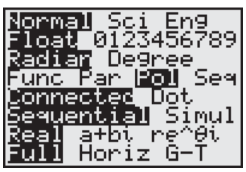

Figure A.63

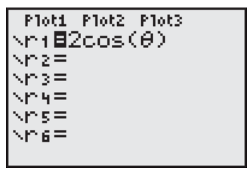

Figure A.64

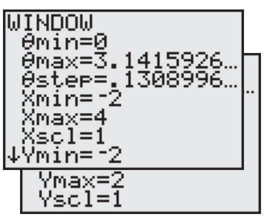

Figure A.65

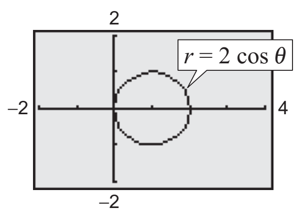

Figure A.66

Sequence Mode To graph the first five terms of a sequence such as $a_n = 4n - 5$, use the *sequence* mode, as shown in Figure A.67. Then enter the sequence in the *equation editor*, as shown in Figure A.68 (assume that n begins with 1). Using the viewing window shown in Figure A.69, you obtain the graph shown in Figure A.70.

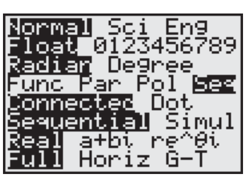

Figure A.67

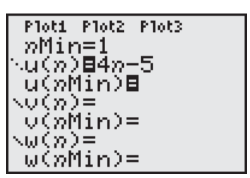

Figure A.68

TECHNOLOGY TIP

Note that when using the different graphing modes of a graphing utility, the utility uses different variables. When the utility is in *function* mode, it uses the variables x and y. In *parametric* mode, the utility uses the variables x, y, and t. In *polar* mode, the utility uses the variables r and θ. In *sequence* mode, the utility uses the variables u (instead of a) and n.

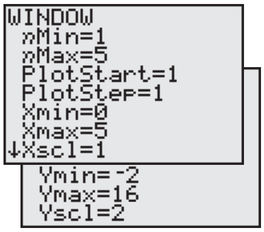

Figure A.69

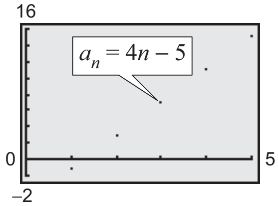

Figure A.70

Connected and Dot Modes

Graphing utilities use the point-plotting method to graph functions. When a graphing utility is in *connected* mode, the utility connects the points that are plotted. When the utility is in *dot* mode, it does not connect the points that are plotted. For example, the graph of $y = x^3$ in *connected* mode is shown in Figure A.71. To graph this function using *dot* mode, first change the mode to *dot* mode (see Figure A.72) and then graph the equation, as shown in Figure A.73. As you can see in Figure A.73, the graph is a collection of dots.

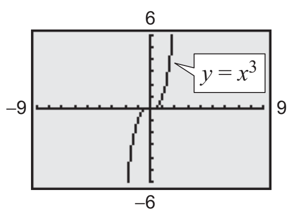

Figure A.71

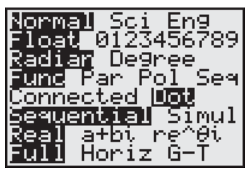

Figure A.72

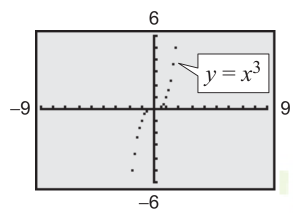

Figure A.73

A problem arises in some graphing utilities when the *connected* mode is used. Graphs with vertical asymptotes, such as rational functions and tangent functions, appear to be connected. For instance, the graph of

$$y = \frac{1}{x + 3}$$

is shown in Figure A.74. Notice how the two portions of the graph appear to be connected with a vertical line at $x = -3$. From your study of rational functions, you know that the graph has a vertical asymptote at $x = -3$ and therefore is undefined when $x = -3$. When using a graphing utility to graph rational functions and other functions that have vertical asymptotes, you should use the *dot* mode to eliminate extraneous vertical lines. Because the *dot* mode of a graphing utility displays a graph as a collection of dots rather than as a smooth curve, in this text, a blue or light red curve is placed behind the graphing utility's display to indicate where the graph should appear, as shown in Figure A.75.

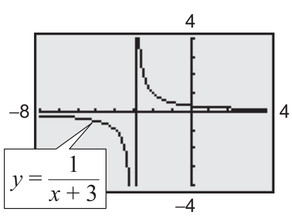

Figure A.74

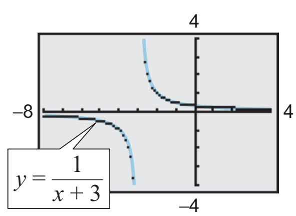

Figure A.75

$_nC_r$ Feature

The $_nC_r$ feature calculates binomial coefficients and the number of combinations of n elements taken r at a time. For example, to find the number of combinations of eight elements taken five at a time, enter 8 (the n-value) on the home screen and choose the $_nC_r$ feature from the *probability* menu of the *math* feature (see Figure A.76). Next, enter 5 (the r-value) on the home screen and press ENTER. You should obtain 56, as shown in Figure A.77.

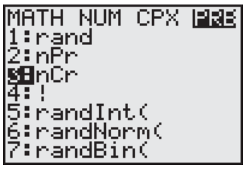

Figure A.76

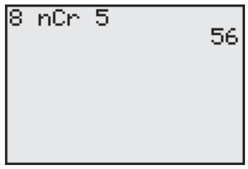

Figure A.77

$_nP_r$ Feature

The $_nP_r$ feature calculates the number of permutations of n elements taken r at a time. For example, to find the number of permutations of six elements taken four at a time, enter 6 (the n-value) on the home screen and choose the $_nP_r$ feature from the *probability* menu of the *math* feature (see Figure A.78). Next enter 4 (the r-value) on the home screen and press ENTER. You should obtain 360, as shown in Figure A.79.

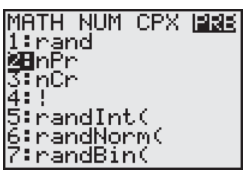

Figure A.78

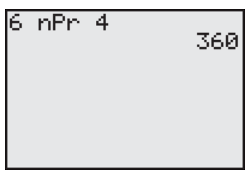

Figure A.79

One-Variable Statistics Feature

Graphing utilities are useful in calculating statistical values for a set of data. The *one-variable statistics* feature analyzes data with one measured variable. This feature outputs the mean of the data, the sum of the data, the sum of the data squared, the sample standard deviation of the data, the population standard deviation of the data, the number of data points, the minimum data value, the maximum data value, the first quartile of the data, the median of the data, and the third quartile of the data. Consider the following data, which shows the hourly earnings (in dollars) for 12 retail sales associates.

5.95, 8.15, 6.35, 7.05, 6.80, 6.10, 7.15, 8.20, 6.50, 7.50, 7.95, 9.25

You can use the *one-variable statistics* feature to determine the mean and standard deviation of the data. First, enter the data in the *list editor*, as shown in Figure A.80. Then choose the *one-variable statistics* feature from the *calculate* menu of the *statistics* feature, as shown in Figure A.81. When using this feature, you must enter a list. In this case, the list is L_1. In Figure A.82, you can see

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that the mean of the data is $\bar{x} \approx 7.25$ and the standard deviation of the data is $\sigma x \approx 0.95$.

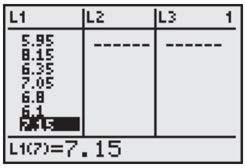


Figure A.80

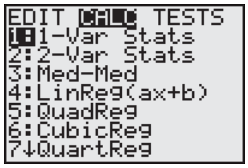


Figure A.81

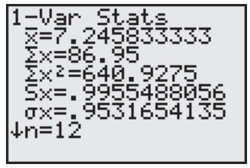


Figure A.82

Regression Feature

Throughout the text, you are asked to use the *regression* feature of a graphing utility to find models for sets of data. Most graphing utilities have built-in regression programs for the following.

Regression	*Form of Model*
Linear	$y = ax + b$ or $y = a + bx$
Quadratic	$y = ax^2 + bx + c$
Cubic	$y = ax^3 + bx^2 + cx + d$
Quartic	$y = ax^4 + bx^3 + cx^2 + dx + e$
Logarithmic	$y = a + b\ln(x)$
Exponential	$y = ab^x$
Power	$y = ax^b$
Logistic	$y = \dfrac{c}{1 + ae^{-bx}}$
Sine	$y = a\sin(bx + c) + d$

For example, you can find a linear model for the numbers y of television sets (in millions) in U.S. households in the years 1996 through 2005, shown in the table. (Source: Television Bureau of Advertising, Inc.)

Year	Number, y
1996	222.8
1997	228.7
1998	235.0
1999	240.3
2000	245.0
2001	248.2
2002	254.4
2003	260.2
2004	268.3
2005	287.0

First, let x represent the year, with $x = 6$ corresponding to 1996. Then enter the data in the *list editor*, as shown in Figure A.83. Note that L_1 contains the years and L_2 contains the numbers of television sets that correspond to the years. Now choose the *linear regression* feature from the *calculate* menu of the *statistics* feature, as shown in Figure A.84. In Figure A.85, you can see that a linear model for the data is given by $y = 6.22x + 183.7$.

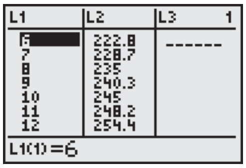


Figure A.83

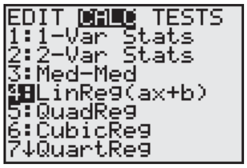


Figure A.84

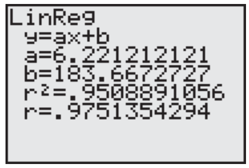


Figure A.85

When you use the *regression* feature of a graphing utility, you will notice that the program may also output an "r-value." (For some calculators, make sure you select the *diagnostics on* feature before you use the *regression* feature. Otherwise, the calculator will not output an r-value.) The r-value or *correlation coefficient* measures how well the linear model fits the data. The closer the value of $|r|$ is to 1, the better the fit. For the data above, $r \approx 0.97514$, which implies that the model is a good fit for the data.

STUDY TIP

In this text, when regression models are found, the number of decimal places in the constant term of the model is the same as the number of decimal places in the data, and then the number of decimal places increases by 1 for each term of increasing power of the independent variable.

Row-Echelon and Reduced Row-Echelon Features

Some graphing utilities have features that can automatically transform a matrix to row-echelon form and reduced row-echelon form. These features can be used to check your solutions to systems of equations.

Row-Echelon Feature

Consider the system of equations and the corresponding augmented matrix shown below.

Linear System

$$\begin{cases} 2x + 5y - 3z = 4 \\ 4x + y = 2 \\ -x + 3y - 2z = -1 \end{cases}$$

Augmented Matrix

$$\begin{bmatrix} 2 & 5 & -3 & \vdots & 4 \\ 4 & 1 & 0 & \vdots & 2 \\ -1 & 3 & -2 & \vdots & -1 \end{bmatrix}$$

You can use the *row-echelon* feature of a graphing utility to write the augmented matrix in row-echelon form. First, enter the matrix in the graphing utility using the *matrix editor*, as shown in Figure A.86. Next, choose the *row-echelon* feature from the *math* menu of the *matrix* feature, as shown in Figure A.87. When using this feature, you must enter the name of the matrix. In this case, the name of the matrix is [A]. You should obtain the matrix shown in Figure A.88. You may have to scroll to the right in order to see all the entries of the matrix.

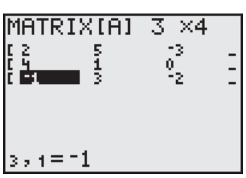


Figure A.86

Figure A.87

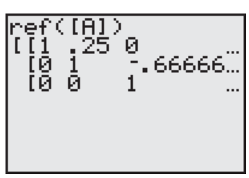


Figure A.88

Reduced Row-Echelon Feature

To write the augmented matrix in reduced row-echelon form, follow the same steps used to write a matrix in row-echelon form except choose the *reduced row-echelon* feature, as shown in Figure A.89. You should obtain the matrix shown in Figure A.90. From Figure A.90, you can conclude that the solution to the system is $x = 3$, $y = -10$, and $z = -16$.

Figure A.89

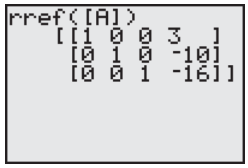


Figure A.90

Sequence Feature

The *sequence* feature is used to display the terms of sequences. For instance, to determine the first five terms of the arithmetic sequence

$$a_n = 3n + 5 \qquad \text{Assume } n \text{ begins with 1.}$$

set the graphing utility to *sequence* mode. Then choose the *sequence* feature from the *operations* menu of the *list* feature, as shown in Figure A.91. When using this feature, you must enter the sequence, the variable (in this case n), the beginning value (in this case 1), and the end value (in this case 5). The first five terms of the sequence are 8, 11, 14, 17, and 20, as shown in Figure A.92. You may have to scroll to the right in order to see all the terms of the sequence.

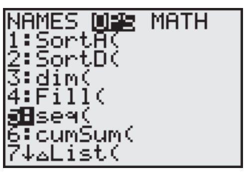


Figure A.91

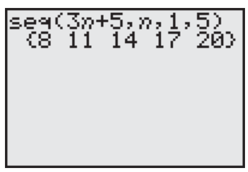


Figure A.92

Shade Feature

Most graphing utilities have a *shade* feature that can be used to graph inequalities. For instance, to graph the inequality $y \le 2x - 3$, first enter the equation $y = 2x - 3$ in the *equation editor*, as shown in Figure A.93. Next, using a standard viewing window (see Figure A.94), graph the equation, as shown in Figure A.95.

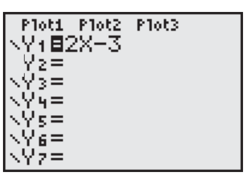


Figure A.93

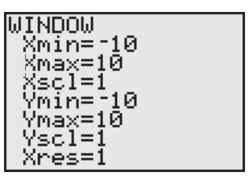


Figure A.94

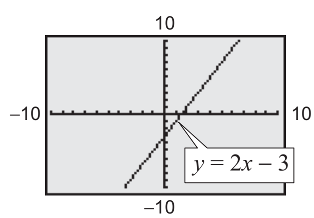


Figure A.95

Because the inequality sign is ≤, you want to shade the region below the line $y = 2x - 3$. Choose the *shade* feature from the *draw* feature menu, as shown in Figure A.96. You must enter a lower function and an upper function. In this case, the lower function is -10 (this is the smallest y-value in the viewing window) and the upper function is Y_1 $(y = 2x - 3)$, as shown in Figure A.97. Then press ENTER to obtain the graph shown in Figure A.98.

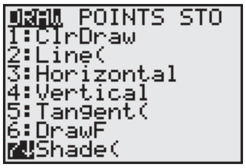


Figure A.96

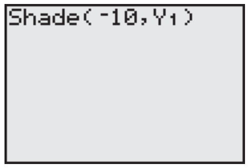


Figure A.97

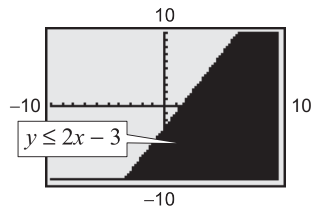


Figure A.98

If you wanted to graph the inequality $y \geq 2x - 3$ (using a standard viewing window), you would enter the lower function as Y_1 $(y = 2x - 3)$ and the upper function as 10 (the largest y-value in the viewing window).

Statistical Plotting Feature

The *statistical plotting* feature plots data that is stored in lists. Most graphing utilities can display the following types of plots.

Plot Type	*Variables*
Scatter plot	x-list, y-list
xy line graph	x-list, y-list
Histogram	x-list, frequency
Box-and-whisker plot	x-list, frequency
Normal probability plot	data list, data axis

For example, use a box-and-whisker plot to represent the following set of data. Then use the graphing utility plot to find the smallest number, the lower quartile, the median, the upper quartile, and the largest number.

17, 19, 21, 27, 29, 30, 37, 27, 15, 23, 19, 16

First, use the *list editor* to enter the values in a list, as shown in Figure A.99. Then go to the *statistical plotting editor*. In this editor you will turn the plot on, select the box-and-whisker plot, select the list you entered in the *list editor*, and enter the frequency of each item in the list, as shown in Figure A.100. Now use the *zoom* feature and choose the *zoom stat* option to set the viewing window and plot the graph, as shown in Figure A.101. Use the *trace* feature to find that the smallest number is 15, the lower quartile is 18, the median is 22, the upper quartile is 28, and the largest number is 37.

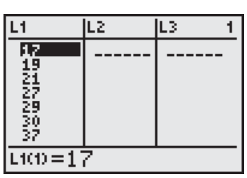


Figure A.99

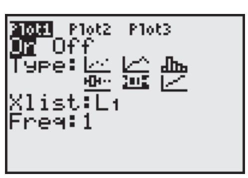


Figure A.100

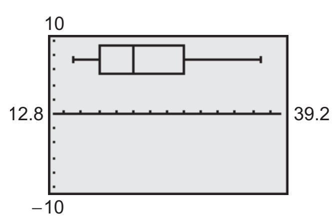


Figure A.101

Sum Feature

The *sum* feature finds the sum of a list of data. For instance, the data below represents a student's quiz scores on 10 quizzes throughout an algebra course.

22, 23, 19, 24, 20, 15, 25, 21, 18, 24

To find the total quiz points the student earned, enter the data in the *list editor*, as shown in Figure A.102. To find the sum, choose the *sum* feature from the *math* menu of the *list* feature, as shown in Figure A.103. You must enter a list. In this case the list is L_1. You should obtain a sum of 211, as shown in Figure A.104.

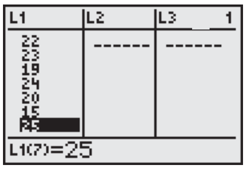


Figure A.102

Figure A.103

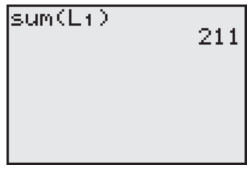


Figure A.104

Sum Sequence Feature

The *sum* feature and the *sequence* feature can be combined to find the sum of a sequence or series. For example, to find the sum

$$\sum_{k=0}^{10} 5^{k+1}$$

first choose the *sum* feature from the *math* menu of the *list* feature, as shown in Figure A.105. Then choose the *sequence* feature from the *operations* menu of the *list* feature, as shown in Figure A.106. You must enter an expression for the sequence, a variable, the lower limit of summation, and the upper limit of summation. After pressing ENTER, you should obtain the sum 61,035,155, as shown in Figure A.107.

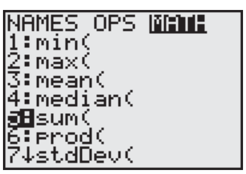


Figure A.105

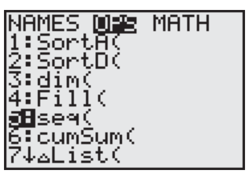


Figure A.106

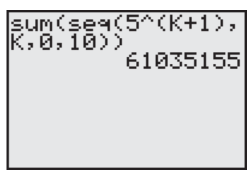


Figure A.107

Table Feature

Most graphing utilities are capable of displaying a table of values with x-values and one or more corresponding y-values. These tables can be used to check solutions of an equation and to generate ordered pairs to assist in graphing an equation by hand.

To use the *table* feature, enter an equation in the *equation editor*. The table may have a setup screen, which allows you to select the starting x-value and the table step or x-increment. You may then have the option of automatically generating values for x and y or building your own table using the *ask* mode (see Figure A.108).

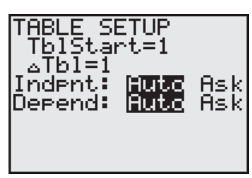


Figure A.108

For example, enter the equation

$$y = \frac{3x}{x + 2}$$

in the *equation editor*, as shown in Figure A.109. In the table setup screen, set the table to start at $x = -4$ and set the table step to 1, as shown in Figure A.110. When you view the table, notice that the first x-value is -4 and that each value after it increases by 1. Also notice that the Y_1 column gives the resulting y-value for each x-value, as shown in Figure A.111. The table shows that the y-value for $x = -2$ is ERROR. This means that the equation is undefined when $x = -2$.

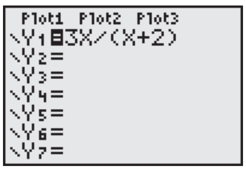


Figure A.109

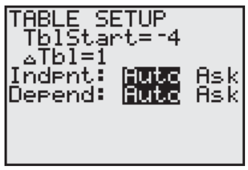


Figure A.110

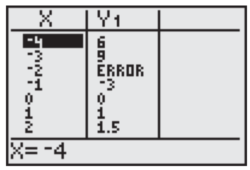


Figure A.111

With the same equation in the *equation editor*, set the independent variable in the table to *ask* mode, as shown in Figure A.112. In this mode, you do not need to set the starting x-value or the table step because you are entering any value you choose for x. You may enter any real value for x—integers, fractions, decimals, irrational numbers, and so forth. If you enter $x = 1 + \sqrt{3}$, the graphing utility may rewrite the number as a decimal approximation, as shown in Figure A.113. You can continue to build your own table by entering additional x-values in order to generate y-values, as shown in Figure A.114.

Figure A.112

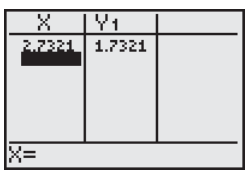


Figure A.113

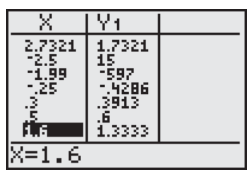


Figure A.114

If you have several equations in the *equation editor*, the table may generate y-values for each equation.

Tangent Feature

Some graphing utilities have the capability of drawing a tangent line to a graph at a given point. For instance, consider the equation

$$y = -x^3 + x + 2.$$

To draw the line tangent to the point $(1, 2)$ on the graph of y, enter the equation in the *equation editor*, as shown in Figure A.115. Using the viewing window shown in Figure A.116, graph the equation, as shown in Figure A.117. Next, choose the *tangent* feature from the *draw* feature menu, as shown in Figure A.118. You can either move the cursor to select a point or enter the x-value at which you want the tangent line to be drawn. Because you want the tangent line to the point $(1, 2)$, enter 1 (see Figure A.119) and then press ENTER. The x-value you entered and the equation of the tangent line are displayed at the bottom of the window, as shown in Figure A.120.

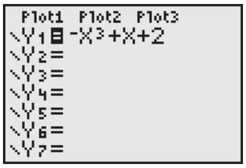


Figure A.115

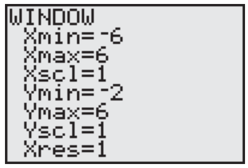


Figure A.116

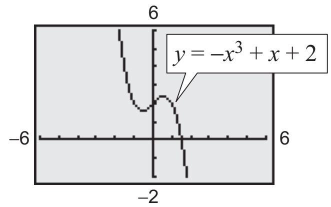


Figure A.117

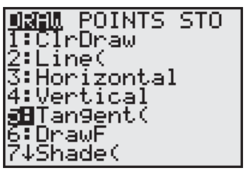


Figure A.118

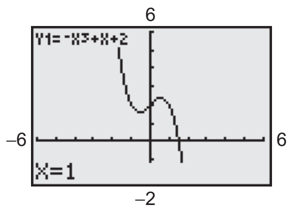


Figure A.119

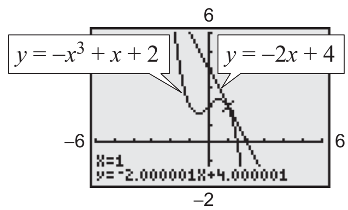


Figure A.120

Trace Feature

For instructions on how to use the *trace* feature, see Zoom and Trace Features on page A23.

Value Feature

The *value* feature finds the value of a function y for a given x-value. To find the value of a function such as $f(x) = 0.5x^2 - 1.5x$ at $x = 1.8$, first enter the function in the *equation editor* (see Figure A.121) and then graph the function (using a standard viewing window), as shown in Figure A.122. Next, choose the *value* feature from the *calculate* menu, as shown in Figure A.123. You will see "X= " displayed at the bottom of the window. Enter the x-value, in this case $x = 1.8$, as shown in Figure A.124. When entering an x-value, be sure it is between the Xmin and Xmax values you entered for the viewing window. Then press ENTER. In Figure A.125, you can see that when $x = 1.8$, $y = -1.08$.

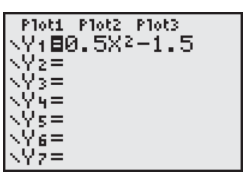


Figure A.121

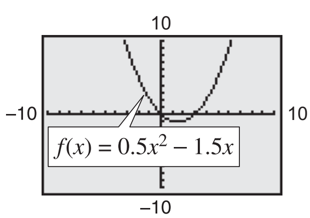


Figure A.122

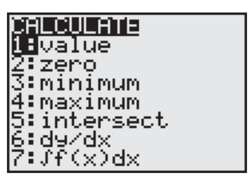


Figure A.123

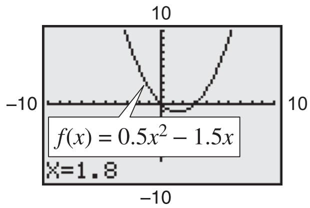


Figure A.124

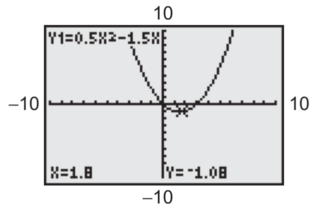


Figure A.125

Viewing Window

A viewing window for a graph is a rectangular portion of the coordinate plane. A viewing window is determined by the following six values (see Figure A.126).

Xmin = the smallest value of x
Xmax = the largest value of x
Xscl = the number of units per tick mark on the x-axis
Ymin = the smallest value of y
Ymax = the largest value of y
Yscl = the number of units per tick mark on the y-axis

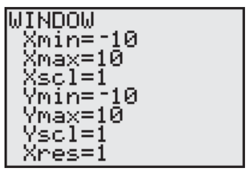


Figure A.126

When you enter these six values in a graphing utility, you are setting the viewing window. On some graphing utilities there is a seventh value for the viewing window labeled Xres. This sets the pixel resolution (1 through 8). For instance, when Xres = 1, functions are evaluated and graphed at each pixel on the x-axis. Some graphing utilities have a standard viewing window, as shown in Figure A.127. To initialize the standard viewing window quickly, choose the *standard viewing window* feature from the *zoom* feature menu (see page A23), as shown in Figure A.128.

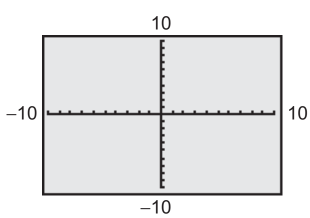


Figure A.127

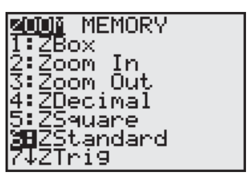


Figure A.128

By choosing different viewing windows for a graph, it is possible to obtain different impressions of the graph's shape. For instance, Figure A.129 shows four different viewing windows for the graph of $y = 0.1x^4 - x^3 + 2x^2$. Of these viewing windows, the one shown in part (a) is the most complete.

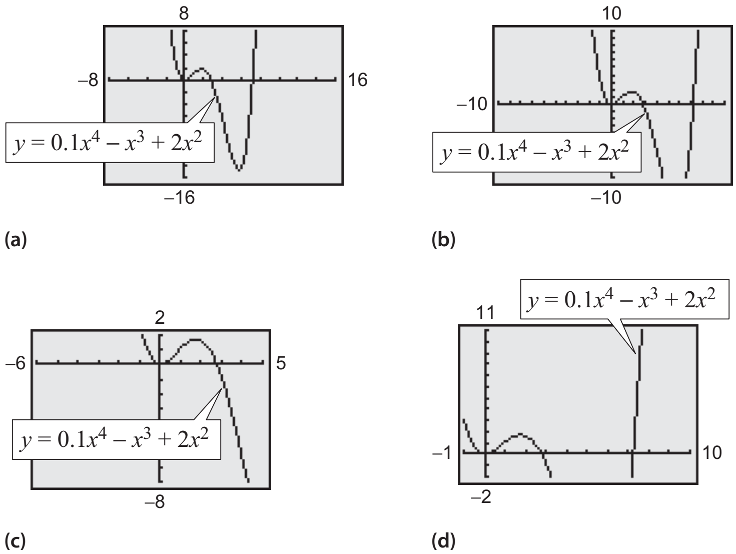


Figure A.129

On most graphing utilities, the display screen is two-thirds as high as it is wide. On such screens, you can obtain a graph with a true geometric perspective by using a square setting—one in which

$$\frac{\text{Ymax} - \text{Ymin}}{\text{Xmax} - \text{Xmin}} = \frac{2}{3}.$$

One such setting is shown in Figure A.130. Notice that the x and y tick marks are equally spaced on a square setting, but not on a standard setting (see Figure A.127). To initialize the square viewing window quickly, choose the *square viewing window* feature from the *zoom* feature menu (see page A23), as shown in Figure A.131.

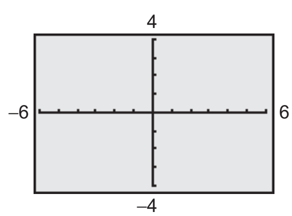


Figure A.130

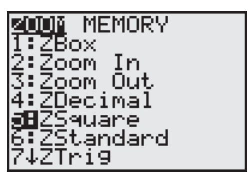


Figure A.131

To see how the viewing window affects the geometric perspective, graph the semicircles $y_1 = \sqrt{9 - x^2}$ and $y_2 = -\sqrt{9 - x^2}$ using a standard viewing window, as shown in Figure A.132. Notice how the circle appears elliptical rather than circular. Now graph y_1 and y_2 using a square viewing window, as shown in Figure A.133. Notice how the circle appears circular. (Note that when you graph the two semicircles, your graphing utility may not connect them. This is because some graphing utilities are limited in their resolution. So, in this text, a blue or light red curve is placed behind the graphing utility's display to indicate where the graph should appear.)

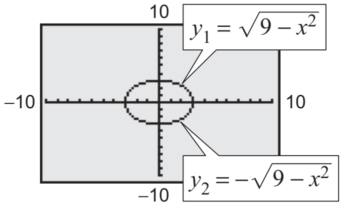


Figure A.132

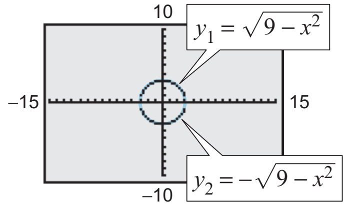


Figure A.133

Zero or Root Feature

The *zero* or *root* feature finds the real zeros of the various types of functions studied in this text. To find the zeros of a function such as

$$f(x) = 2x^3 - 4x$$

first enter the function in the *equation editor*, as shown in Figure A.134. Now graph the equation (using a standard viewing window), as shown in Figure A.135. From the graph you can see that the graph of the function crosses the x-axis three times, so the function has three zeros.

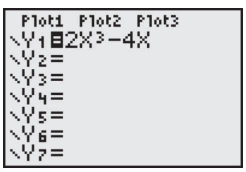


Figure A.134

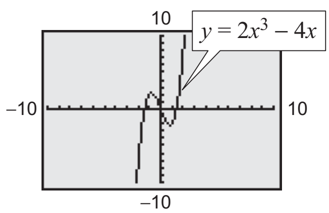


Figure A.135

To find these zeros, choose the *zero* feature found in the *calculate* menu (see Figure A.136). Next, trace the cursor along the graph to a point left of one of the zeros and press ENTER (see Figure A.137). Then trace the cursor along the graph to a point right of the zero and press ENTER (see Figure A.138). Note the two arrows near the top of the display marking the left and right bounds, as shown in Figure A.139. Now trace the cursor along the graph between the two bounds and as close to the zero as you can (see Figure A.140) and press ENTER. In Figure A.141, you can see that one zero of the function is $x \approx -1.414214$.

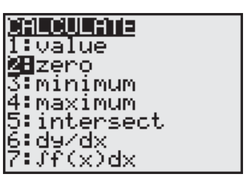


Figure A.136

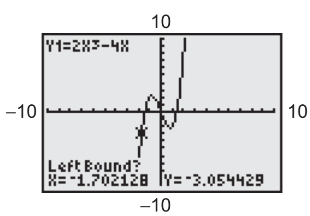


Figure A.137

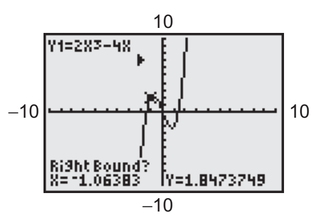


Figure A.138

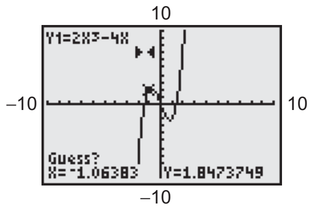


Figure A.139

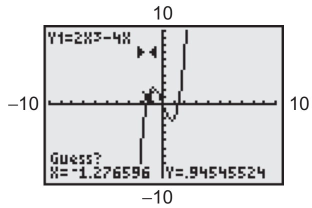


Figure A.140

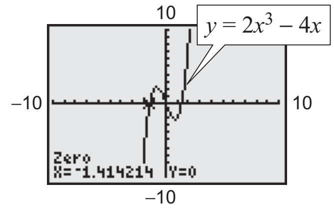


Figure A.141

Repeat this process to determine that the other two zeros of the function are $x = 0$ (see Figure A.142) and $x \approx 1.414214$ (see Figure A.143).

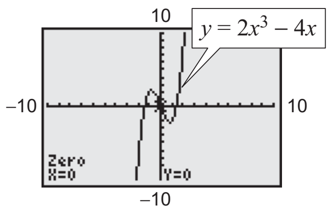


Figure A.142

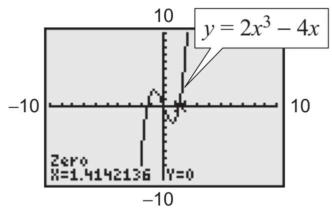


Figure A.143

Zoom and Trace Features

The *zoom* feature enables you to adjust the viewing window of a graph quickly (see Figure A.144). For example, the *zoom box* feature allows you to create a new viewing window by drawing a box around any part of the graph.

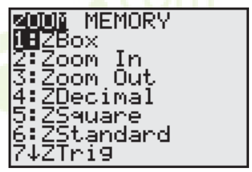


Figure A.144

The *trace* feature moves from point to point along a graph. For instance, enter the equation $y = 2x^3 - 3x + 2$ in the *equation editor* (see Figure A.145) and graph the equation, as shown in Figure A.146. To activate the *trace* feature, press TRACE; then use the arrow keys to move the cursor along the graph. As you trace the graph, the coordinates of each point are displayed, as shown in Figure A.147.

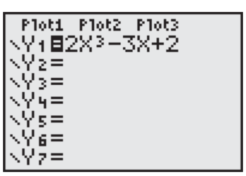


Figure A.145

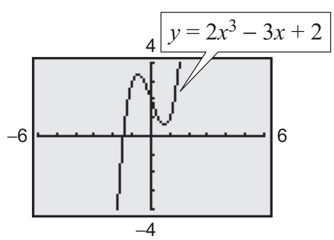


Figure A.146

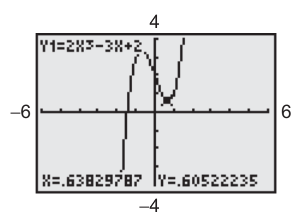


Figure A.147

The *trace* feature combined with the *zoom* feature enables you to obtain better and better approximations of desired points on a graph. For instance, you can use the *zoom* feature to approximate the x-intercept of the graph of $y = 2x^3 - 3x + 2$. From the viewing window shown in Figure A.146, the graph appears to have only one x-intercept. This intercept lies between -2 and -1. To zoom in on the x-intercept, choose the *zoom-in* feature from the *zoom* feature menu, as shown in Figure A.148. Next, trace the cursor to the point you want to zoom in on, in this case the x-intercept (see Figure A.149). Then press

ENTER. You should obtain the graph shown in Figure A.150. Now, using the *trace* feature, you can approximate the x-intercept to be $x \approx -1.468085$, as shown in Figure A.151. Use the *zoom-in* feature again to obtain the graph shown in Figure A.152. Using the *trace* feature, you can approximate the x-intercept to be $x \approx -1.476064$, as shown in Figure A.153.

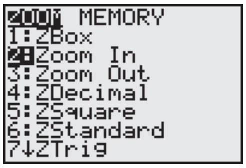


Figure A.148

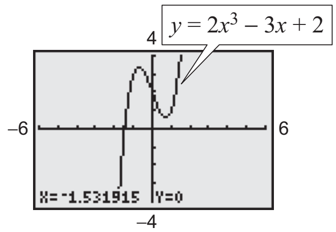


Figure A.149

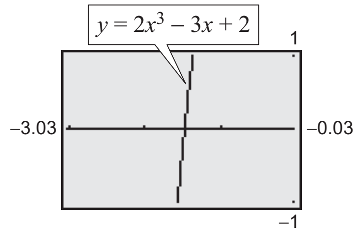


Figure A.150

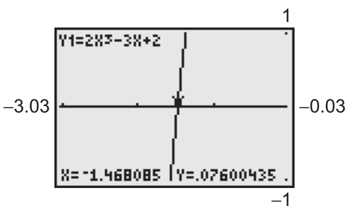


Figure A.151

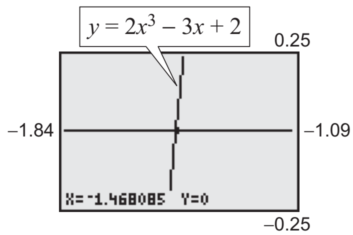


Figure A.152

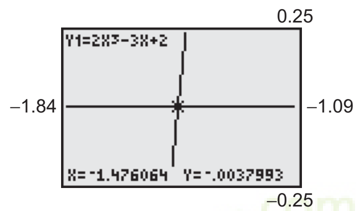


Figure A.153

Here are some suggestions for using the *zoom* feature.

1. With each successive zoom-in, adjust the scale so that the viewing window shows at least one tick mark on each side of the x-intercept.
2. The error in your approximation will be less than the distance between two scale marks.
3. The *trace* feature can usually be used to add one more decimal place of accuracy without changing the viewing window.

You can adjust the scale in Figure A.153 to obtain a better approximation of the x-intercept. Using the suggestions above, change the viewing window settings so that the viewing window shows at least one tick mark on each side of the x-intercept, as shown in Figure A.154. From Figure A.154, you can determine that the error in your approximation will be less than 0.001 (the Xscl value). Then, using the *trace* feature, you can improve the approximation, as shown in Figure A.155. To three decimal places, the x-intercept is $x \approx -1.476$.

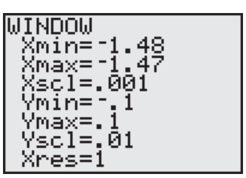


Figure A.154

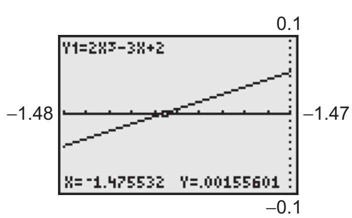


Figure A.155

Appendix B: Concepts in Statistics

B.1 Measures of Central Tendency and Dispersion

Mean, Median, and Mode

In many real-life situations, it is helpful to describe data by a single number that is most representative of the entire collection of numbers. Such a number is called a **measure of central tendency.** The most commonly used measures are as follows.

1. The **mean,** or **average,** of n numbers is the sum of the numbers divided by n.
2. The **median** of n numbers is the middle number when the numbers are written in numerical order. If n is even, the median is the average of the two middle numbers.
3. The **mode** of n numbers is the number that occurs most frequently. If two numbers tie for most frequent occurrence, the collection has two modes and is called **bimodal.**

What you should learn

- Find and interpret the mean, median, and mode of a set of data.
- Determine the measure of central tendency that best represents a set of data.
- Find the standard deviation of a set of data.
- Use box-and-whisker plots.

Why you should learn it

Measures of central tendency and dispersion provide a convenient way to describe and compare sets of data. For instance, in Exercise 34 on page A33, the mean and standard deviation are used to analyze the prices of gold for the years 1982 through 2005.

Example 1 Comparing Measures of Central Tendency

On an interview for a job, the interviewer tells you that the average annual income of the company's 25 employees is \$60,849. The actual annual incomes of the 25 employees are shown below. What are the mean, median, and mode of the incomes?

\$17,305,	\$478,320,	\$45,678,	\$18,980,	\$17,408,
\$25,676,	\$28,906,	\$12,500,	\$24,540,	\$33,450,
\$12,500,	\$33,855,	\$37,450,	\$20,432,	\$28,956,
\$34,983,	\$36,540,	\$250,921,	\$36,853,	\$16,430,
\$32,654,	\$98,213,	\$48,980,	\$94,024,	\$35,671

Solution

The mean of the incomes is

$$\text{Mean} = \frac{17{,}305 + 478{,}320 + 45{,}678 + 18{,}980 + \cdots + 35{,}671}{25}$$

$$= \frac{1{,}521{,}225}{25} = \$60{,}849.$$

To find the median, order the incomes as follows.

\$12,500,	\$12,500,	\$16,430,	\$17,305,	\$17,408,
\$18,980,	\$20,432,	\$24,540,	\$25,676,	\$28,906,
\$28,956,	\$32,654,	\$33,450,	\$33,855,	\$34,983,
\$35,671,	\$36,540,	\$36,853,	\$37,450,	\$45,678,
\$48,980,	\$94,024,	\$98,213,	\$250,921,	\$478,320

From this list, you can see that the median income is \$33,450. You can also see that \$12,500 is the only income that occurs more than once. So, the mode is \$12,500.

CHECKPOINT Now try Exercise 1.

In Example 1, was the interviewer telling you the truth about the annual incomes? Technically, the person was telling the truth because the average is (generally) defined to be the mean. However, of the three measures of central tendency—*mean:* \$60,849, *median:* \$33,450, *mode:* \$12,500—it seems clear that the median is most representative. The mean is inflated by the two highest salaries.

Choosing a Measure of Central Tendency

Which of the three measures of central tendency is most representative of a particular data set? The answer is that it depends on the distribution of the data *and* the way in which you plan to use the data.

For instance, in Example 1, the mean salary of \$60,849 does not seem very representative to a potential employee. To a city income tax collector who wants to estimate 1% of the total income of the 25 employees, however, the mean is precisely the right measure.

Example 2 Choosing a Measure of Central Tendency

Which measure of central tendency is most representative of the data given in each frequency distribution?

a.

Number	1	2	3	4	5	6	7	8	9
Frequency	7	20	15	11	8	3	2	0	15

b.

Number	1	2	3	4	5	6	7	8	9
Frequency	9	8	7	6	5	6	7	8	9

c.

Number	1	2	3	4	5	6	7	8	9
Frequency	6	1	2	3	5	5	8	3	0

Solution

a. For this data set, the mean is 4.23, the median is 3, and the mode is 2. Of these, the median or mode is probably the most representative measure.

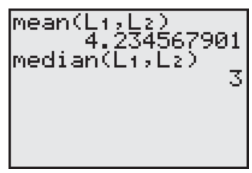
b. For this data set, the mean and median are each 5 and the modes are 1 and 9 (the distribution is bimodal). Of these, the mean or median is the most representative measure.

c. For this data set, the mean is 4.59, the median is 5, and the mode is 7. Of these, the mean or median is the most representative measure.

CHECKPOINT Now try Exercise 15.

TECHNOLOGY TIP

Calculating the mean and median of a large data set can become time consuming. Most graphing utilities have *mean* and *median* features that can be used to find the means and medians of data sets. Enter the data from Example 2(a) in the *list editor* of a graphing utility. Then use the *mean* and *median* features to verify the solution to Example 2(a), as shown below.



For instructions on how to use the *list* feature, the *mean* feature, and the *median* feature, see Appendix A; for specific keystrokes, go to this textbook's *Online Study Center*.

Variance and Standard Deviation

Very different sets of numbers can have the same mean. You will now study two **measures of dispersion,** which give you an idea of how much the numbers in a set differ from the mean of the set. These two measures are called the *variance* of the set and the *standard deviation* of the set.

Definitions of Variance and Standard Deviation

Consider a set of numbers $\{x_1, x_2, \ldots, x_n\}$ with a mean of $\bar{x}$. The **variance** of the set is

$$v = \frac{(x_1 - \bar{x})^2 + (x_2 - \bar{x})^2 + \cdots + (x_n - \bar{x})^2}{n}$$

and the **standard deviation** of the set is $\sigma = \sqrt{v}$ (σ is the lowercase Greek letter *sigma*).

The standard deviation of a set is a measure of how much a typical number in the set differs from the mean. The greater the standard deviation, the more the numbers in the set vary from the mean. For instance, each of the following sets has a mean of 5.

$$\{5, 5, 5, 5\}, \qquad \{4, 4, 6, 6\}, \qquad \text{and} \qquad \{3, 3, 7, 7\}$$

The standard deviations of the sets are 0, 1, and 2.

$$\sigma_1 = \sqrt{\frac{(5-5)^2 + (5-5)^2 + (5-5)^2 + (5-5)^2}{4}} = 0$$

$$\sigma_2 = \sqrt{\frac{(4-5)^2 + (4-5)^2 + (6-5)^2 + (6-5)^2}{4}} = 1$$

$$\sigma_3 = \sqrt{\frac{(3-5)^2 + (3-5)^2 + (7-5)^2 + (7-5)^2}{4}} = 2$$

Example 3 Estimations of Standard Deviation

Consider the three frequency distributions represented by the bar graphs in Figure B.1. Which set has the smallest standard deviation? Which has the largest?

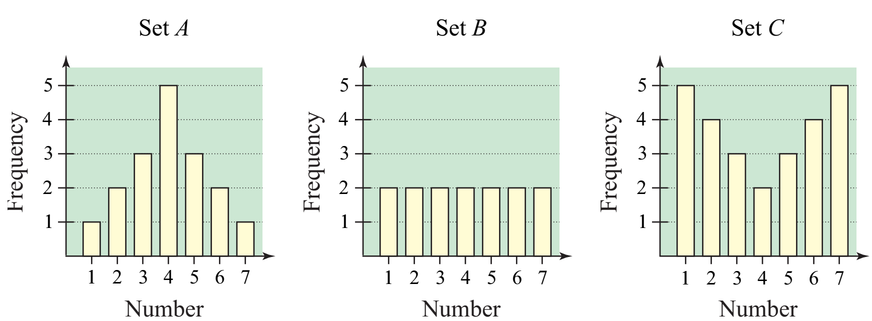


Figure B.1

Solution

Of the three sets, the numbers in set A are grouped most closely to the center and the numbers in set C are the most dispersed. So, set A has the smallest standard deviation and set C has the largest standard deviation.

CHECKPOINT Now try Exercise 17.

STUDY TIP

In Example 3, you may find it helpful to write each set numerically. For instance, set A is

$$\{1, 2, 2, 3, 3, 3, 4, 4, 4, 4, 5, 5, 5, 6, 6, 7\}.$$

Example 4 Finding a Standard Deviation

Find the standard deviation of each set shown in Example 3.

Solution

Because of the symmetry of each bar graph, you can conclude that each has a mean of $\bar{x} = 4$. The standard deviation of set A is

$$\sigma = \sqrt{\frac{(-3)^2 + 2(-2)^2 + 3(-1)^2 + 5(0)^2 + 3(1)^2 + 2(2)^2 + (3)^2}{17}}$$

$$\approx 1.53.$$

The standard deviation of set B is

$$\sigma = \sqrt{\frac{2(-3)^2 + 2(-2)^2 + 2(-1)^2 + 2(0)^2 + 2(1)^2 + 2(2)^2 + 2(3)^2}{14}}$$

$$= 2.$$

The standard deviation of set C is

$$\sigma = \sqrt{\frac{5(-3)^2 + 4(-2)^2 + 3(-1)^2 + 2(0)^2 + 3(1)^2 + 4(2)^2 + 5(3)^2}{26}}$$

$$\approx 2.22.$$

These values confirm the results of Example 3. That is, set A has the smallest standard deviation and set C has the largest.

CHECKPOINT Now try Exercise 21.

The following alternative formula provides a more efficient way to compute the standard deviation.

Alternative Formula for Standard Deviation

The standard deviation of $\{x_1, x_2, . . . , x_n\}$ is given by

$$\sigma = \sqrt{\frac{x_1^2 + x_2^2 + \cdot \cdot \cdot + x_n^2}{n} - \bar{x}^2}.$$

Because of lengthy computations, this formula is difficult to verify. Conceptually, however, the process is straightforward. It consists of showing that the expressions

$$\sqrt{\frac{(x_1 - \bar{x})^2 + (x_2 - \bar{x})^2 + \cdot \cdot \cdot + (x_n - \bar{x})^2}{n}}$$

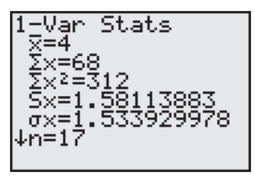
and

$$\sqrt{\frac{x_1^2 + x_2^2 + \cdot \cdot \cdot + x_n^2}{n} - \bar{x}^2}$$

are equivalent. Try verifying this equivalence for the set $\{x_1, x_2, x_3\}$ with $\bar{x} = (x_1 + x_2 + x_3)/3$.

TECHNOLOGY TIP

Calculating the standard deviation of a large data set can become time-consuming. Most graphing utilities have *statistical* features that can be used to find different statistical values of data sets. Enter the data from set A in Example 3 in the *list editor* of a graphing utility. Then use the *one-variable statistics* feature to verify the solution to Example 4, as shown below.



In the figure above, the standard deviation is represented as σx, which is about 1.53. For instructions on how to use the *one-variable statistics* feature, see Appendix A; for specific keystrokes, go to this textbook's *Online Study Center*.

Example 5 Using the Alternative Formula

Use the alternative formula for standard deviation to find the standard deviation of the following set of numbers.

5, 6, 6, 7, 7, 8, 8, 8, 9, 10

Solution

Begin by finding the mean of the set, which is 7.4. So, the standard deviation is

$$\sigma = \sqrt{\frac{5^2 + 2(6^2) + 2(7^2) + 3(8^2) + 9^2 + 10^2}{10} - (7.4)^2}$$

$$= \sqrt{\frac{568}{10} - 54.76} = \sqrt{2.04} \approx 1.43.$$

You can use the *one-variable statistics* feature of a graphing utility to check this result.

CHECKPOINT Now try Exercise 27.

A well-known theorem in statistics, called *Chebychev's Theorem*, states that at least

$$1 - \frac{1}{k^2}$$

of the numbers in a distribution must lie within k standard deviations of the mean. So, at least 75% of the numbers in a collection must lie within two standard deviations of the mean, and at least 88.9% of the numbers must lie within three standard deviations of the mean. For most distributions, these percents are low. For instance, in all three distributions shown in Example 3, 100% of the numbers lie within two standard deviations of the mean.

Example 6 Describing a Distribution

The table at the right shows the number of outpatient visits to hospitals (in millions) in each state and the District of Columbia in 2003. Find the mean and standard deviation of the numbers. What percent of the numbers lie within two standard deviations of the mean? (Source: Health Forum)

AK	1	MT	3
AL	9	NC	15
AR	5	ND	2
AZ	7	NE	4
CA	48	NH	3
CO	7	NJ	15
CT	7	NM	5
DC	2	NV	2
DE	2	NY	48
FL	22	OH	30
GA	13	OK	6
HI	2	OR	8
IA	10	PA	33
ID	3	RI	2
IL	27	SC	7
IN	15	SD	2
KS	6	TN	10
KY	9	TX	32
LA	11	UT	5
MA	20	VA	11
MD	7	VT	2
ME	4	WA	10
MI	27	WI	12
MN	9	WV	6
MO	16	WY	1
MS	4		

Solution

Begin by entering the numbers in a graphing utility. Then use the *one-variable statistics* feature to obtain $\bar{x} \approx 11.12$ and $\sigma \approx 11.10$. The interval that contains all numbers that lie within two standard deviations of the mean is

$$[11.12 - 2(11.10), 11.12 + 2(11.10)] \quad \text{or} \quad [-11.08, 33.32].$$

From the table you can see that all but two of the numbers (96%) lie in this interval—all but the numbers that correspond to the numbers of outpatient visits to hospitals in California and New York.

CHECKPOINT Now try Exercise 35.

Box-and-Whisker Plots

Standard deviation is the measure of dispersion that is associated with the mean. **Quartiles** measure dispersion associated with the median.

> **Definition of Quartiles**
>
> Consider an ordered set of numbers whose median is m. The **lower quartile** is the median of the numbers that occur on or before m. The **upper quartile** is the median of the numbers that occur on or after m.

Example 7 Finding Quartiles of a Set

Find the lower and upper quartiles of the following set.

34, 14, 24, 16, 12, 18, 20, 24, 16, 26, 13, 27

Solution

Begin by ordering the set.

12, 13, 14,	16, 16, 18,	20, 24, 24,	26, 27, 34
1st 25%	2nd 25%	3rd 25%	4th 25%

The median of the entire set is 19. The median of the six numbers that are less than 19 is 15. So, the lower quartile is 15. The median of the six numbers that are greater than 19 is 25. So, the upper quartile is 25.

CHECKPOINT Now try Exercise 37(a).

Quartiles are represented graphically by a **box-and-whisker plot,** as shown in Figure B.2. In the plot, notice that five numbers are listed: the smallest number, the lower quartile, the median, the upper quartile, and the largest number. Also notice that the numbers are spaced proportionally, as though they were on a real number line.

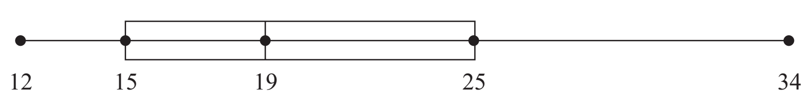


Figure B.2

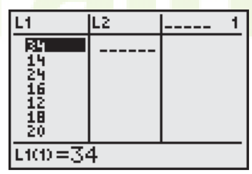


Figure B.3

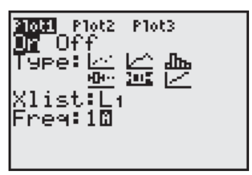


Figure B.4

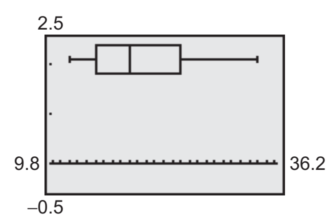


Figure B.5

TECHNOLOGY TIP You can use a graphing utility to graph the box-and-whisker plot in Figure B.2. First enter the data in the graphing utility's *list editor*, as shown in Figure B.3. Then use the *statistical plotting* feature to set up the box-and-whisker plot, as shown in Figure B.4. Finally, display the box-and-whisker plot (using the *ZoomStat* feature), as shown in Figure B.5. For instructions on how to use the *list editor* and the *statistical plotting* features, see Appendix A; for specific keystrokes, go to this textbook's *Online Study Center*.

The next example shows how to find quartiles when the number of elements in a set is not divisible by 4.

Example 8 Sketching Box-and-Whisker Plots

Sketch a box-and-whisker plot for each data set.

a. 82, 82, 83, 85, 87, 89, 90, 94, 95, 95, 96, 98, 99

b. 11, 13, 13, 15, 17, 17, 20, 24, 24, 27

Solution

a. This set has 13 numbers. The median is 90 (the seventh number). The lower quartile is 84 (the median of the first six numbers). The upper quartile is 95.5 (the median of the last six numbers). See Figure B.6.

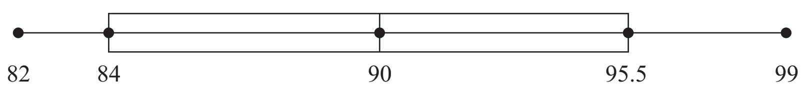


Figure B.6

b. This set has 10 numbers. The median is 17 (the average of the fifth and sixth numbers). The lower quartile is 13 (the median of the first five numbers). The upper quartile is 24 (the median of the last five numbers). See Figure B.7.

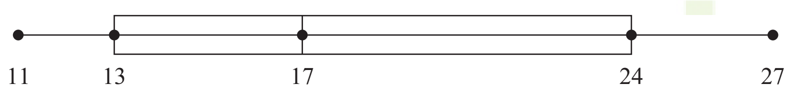


Figure B.7

CHECKPOINT Now try Exercise 37(b).

B.1 Exercises

See www.CalcChat.com for worked-out solutions to odd-numbered exercises.

Vocabulary Check

Fill in the blanks.

1. A single number that is the most representative of a data set is called a _______ of _______ .

2. If two numbers are tied for the most frequent occurrence, the collection has two _______ and is called _______ .

3. Two measures of dispersion are called the ______ and the ______ of a data set.

4. _______ measure dispersion associated with the median.

In Exercises 1–6, find the mean, median, and mode of the data set.

1. 5, 12, 7, 14, 8, 9, 7

2. 30, 37, 32, 39, 33, 34, 32

3. 5, 12, 7, 24, 8, 9, 7

4. 20, 37, 32, 39, 33, 34, 32

5. 5, 12, 7, 14, 9, 7

6. 30, 37, 32, 39, 34, 32

7. ***Reasoning***

(a) Compare your answers in Exercises 1 and 3 with those in Exercises 2 and 4. Which of the measures of central tendency is sensitive to extreme measurements? Explain your reasoning.

(b) Add 6 to each measurement in Exercise 1 and calculate the mean, median, and mode of the revised measurements. How are the measures of central tendency changed?

(c) If a constant k is added to each measurement in a set of data, how will the measures of central tendency change?

8. ***Consumer Awareness*** A person had the following monthly bills for electricity. What are the mean and median of the collection of bills?

January	\$67.92	February	\$59.84
March	\$52.00	April	\$52.50
May	\$57.99	June	\$65.35
July	\$81.76	August	\$74.98
September	\$87.82	October	\$83.18
November	\$65.35	December	\$57.00

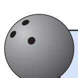
9. ***Car Rental*** A car rental company kept the following record of the numbers of miles a rental car was driven. What are the mean, median, and mode of the data?

Monday	410	Tuesday	260
Wednesday	320	Thursday	320
Friday	460	Saturday	150

10. ***Families*** A study was done on families having six children. The table shows the numbers of families in the study with the indicated numbers of girls. Determine the mean, median, and mode of the data.

Number of girls	0	1	2	3	4	5	6
Frequency	1	24	45	54	50	19	7

11. ***Bowling Scores*** The table shows the bowling scores for a three-game series of a three-member team.



Team member	Game 1	Game 2	Game 3
Jay	181	222	196
Hank	199	195	205
Buck	202	251	235

(a) Find the mean for each team member.

(b) Find the mean for the entire team for the three-game series.

(c) Find the median for the entire team for the three-game series.

12. ***Selling Price*** The selling price of 12 new homes built in one subdivision are listed.

\$525,000	\$375,000	\$425,000	\$550,000
\$385,000	\$500,000	\$550,000	\$425,000
\$475,000	\$500,000	\$350,000	\$450,000

(a) Find the mean, mode, and median of the selling prices.

(b) Which measure of central tendency best describes the prices? Explain.

13. ***Think About It*** Construct a collection of numbers that has the following properties. If this is not possible, explain why.

Mean $= 6$, median $= 4$, mode $= 4$

14. ***Think About It*** Construct a collection of numbers that has the following properties. If this is not possible, explain why.

Mean $= 6$, median $= 6$, mode $= 4$

15. ***Test Scores*** An English professor records the following scores for a 100-point exam.

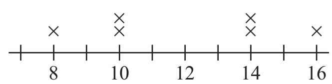
99, 64, 80, 77, 59, 72, 87, 79, 92, 88, 90, 42, 20, 89, 42, 100, 98, 84, 78, 91

Which measure of central tendency best describes these test scores?

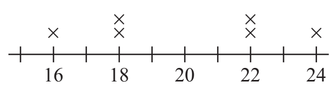
16. ***Shoe Sales*** A salesman sold eight pairs of men's brown dress shoes. The sizes of the eight pairs were as follows: $10\frac{1}{2}$, 8, 12, $10\frac{1}{2}$, 10, $9\frac{1}{2}$, 11, and $10\frac{1}{2}$. Which measure (or measures) of central tendency best describes (describe) the typical shoe size for this data?

In Exercises 17 and 18, line plots of data sets are given. Determine the mean and standard deviation of each set.

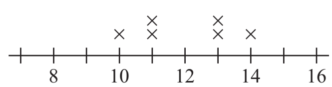
17. (a)



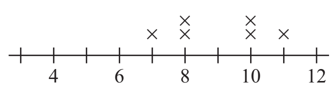
(b)



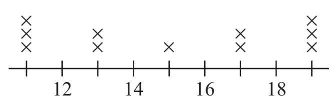
(c)

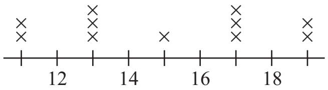


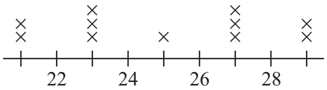
(d)

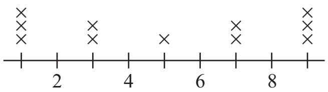


18. (a)



(b)


(c)


(d)


In Exercises 19–26, find the mean ($\bar{x}$), variance (v), and standard deviation (σ) of the set.

19. 4, 10, 8, 2

20. 3, 15, 6, 9, 2

21. 0, 1, 1, 2, 2, 2, 3, 3, 4

22. 2, 2, 2, 2, 2, 2

23. 1, 2, 3, 4, 5, 6, 7

24. 1, 1, 1, 5, 5, 5

25. 49, 62, 40, 29, 32, 70

26. 1.5, 0.4, 2.1, 0.7, 0.8

In Exercises 27–30, use the alternative formula to find the standard deviation of the set.

27. 2, 4, 6, 6, 13, 5

28. 246, 336, 473, 167, 219, 359

29. 8.1, 6.9, 3.7, 4.2, 6.1

30. 9.0, 7.5, 3.3, 7.4, 6.0

31. ***Reasoning*** Without calculating the standard deviation, explain why the set {4, 4, 20, 20} has a standard deviation of 8.

32. ***Reasoning*** If the standard deviation of a set of numbers is 0, what does this imply about the set?

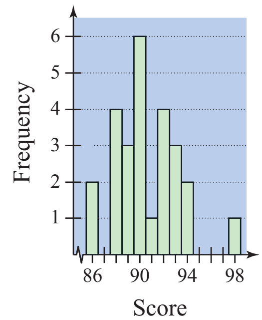
33. ***Test Scores*** An instructor adds five points to each student's exam score. Will this change the mean or standard deviation of the exam scores? Explain.

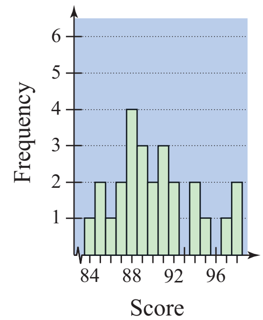
34. ***Price of Gold*** The following data represents the average prices of gold (in dollars per fine ounce) for the years 1982 to 2005. Use a computer or graphing utility to find the mean, variance, and standard deviation of the data. What percent of the data lies within two standard deviations of the mean? (Source: National Mining Association)

376,	424,	361,	317,	368,	447,
437,	381,	384,	362,	344,	360,
384,	384,	388,	331,	294,	279,
279,	271,	310,	363,	410,	445

35. ***Test Scores*** The scores on a mathematics exam given to 600 science and engineering students at a college had a mean and standard deviation of 235 and 28, respectively. Use Chebychev's Theorem to determine the intervals containing at least $\frac{3}{4}$ and at least $\frac{8}{9}$ of the scores. How would the intervals change if the standard deviation were 16?

36. ***Think About It*** The histograms represent the test scores of two classes of a college course in mathematics. Which histogram has the smaller standard deviation?





In Exercises 37–40, (a) find the lower and upper quartiles of the data and (b) sketch a box-and-whisker plot for the data without using a graphing utility.

37. 23, 15, 14, 23, 13, 14, 13, 20, 12

38. 11, 10, 11, 14, 17, 16, 14, 11, 8, 14, 20

39. 46, 48, 48, 50, 52, 47, 51, 47, 49, 53

40. 25, 20, 22, 28, 24, 28, 25, 19, 27, 29, 28, 21

In Exercises 41–44, use a graphing utility to create a box-and-whisker plot for the data.

41. 19, 12, 14, 9, 14, 15, 17, 13, 19, 11, 10, 19

42. 9, 5, 5, 5, 6, 5, 4, 12, 7, 10, 7, 11, 8, 9, 9

43. 20.1, 43.4, 34.9, 23.9, 33.5, 24.1, 22.5, 42.4, 25.7, 17.4, 23.8, 33.3, 17.3, 36.4, 21.8

44. 78.4, 76.3, 107.5, 78.5, 93.2, 90.3, 77.8, 37.1, 97.1, 75.5, 58.8, 65.6

45. ***Product Lifetime*** A company has redesigned a product in an attempt to increase the lifetime of the product. The two sets of data list the lifetimes (in months) of 20 units with the original design and 20 units with the new design. Create a box-and-whisker plot for each set of data, and then comment on the differences between the plots.

Original Design

15.1	78.3	56.3	68.9	30.6
27.2	12.5	42.7	72.7	20.2
53.0	13.5	11.0	18.4	85.2
10.8	38.3	85.1	10.0	12.6

New Design

55.8	71.5	25.6	19.0	23.1
37.2	60.0	35.3	18.9	80.5
46.7	31.1	67.9	23.5	99.5
54.0	23.2	45.5	24.8	87.8

B.2 Least Squares Regression

What you should learn

- Use the sum of squared differences to determine a least squares regression line.
- Find a least squares regression line for a set of data.
- Find a least squares regression parabola for a set of data.

Why you should learn it

The method of least squares provides a way of creating a mathematical model for a set of data, which can then be analyzed.

In many of the examples and exercises in this text, you have been asked to use the *regression* feature of a graphing utility to find mathematical models for sets of data. The *regression* feature of a graphing utility uses the **method of least squares** to find a mathematical model for a set of data. As a measure of how well a model fits a set of data points

$$\{(x_1, y_1), (x_2, y_2), (x_3, y_3), . . . , (x_n, y_n)\}$$

you can add the squares of the differences between the actual y-values and the values given by the model to obtain the **sum of the squared differences.** For instance, the table shows the heights x (in feet) and the diameters y (in inches) of eight trees. The table also shows the values of a linear model $y^* = 0.54x - 29.5$ for each x-value. The sum of squared differences for the model is 51.7.

x	70	72	75	76	85	78	77	80
y	8.3	10.5	11.0	11.4	12.9	14.0	16.3	18.0
y^*	8.3	9.38	11.0	11.54	16.4	12.62	12.08	13.7
$(y - y^*)^2$	0	1.2544	0	0.0196	12.25	1.9044	17.8084	18.49

The model that has the *least* sum of squared differences is the **least squares regression** line for the data. The least squares regression line for the data in the table is $y \approx 0.43x - 20.3$. The sum of squared differences is 43.3.

To find the least squares regression line $y = ax + b$ for the points $\{(x_1, y_1), (x_2, y_2), (x_3, y_3), . . . , (x_n, y_n)\}$ algebraically, you need to solve the following system for a and b.

$$\begin{cases} nb + \left(\sum_{i=1}^{n} x_i\right)a = \sum_{i=1}^{n} y_i \\ \left(\sum_{i=1}^{n} x_i\right)b + \left(\sum_{i=1}^{n} x_i^2\right)a = \sum_{i=1}^{n} x_i y_i \end{cases}$$

In the system,

$$\sum_{i=1}^{n} x_i = x_1 + x_2 + \cdot \cdot \cdot + x_n$$

$$\sum_{i=1}^{n} y_i = y_1 + y_2 + \cdot \cdot \cdot + y_n$$

$$\sum_{i=1}^{n} x_i^2 = x_1^2 + x_2^2 + \cdot \cdot \cdot + x_n^2$$

$$\sum_{i=1}^{n} x_i y_i = x_1 y_1 + x_2 y_2 + \cdot \cdot \cdot + x_n y_n.$$

TECHNOLOGY SUPPORT

For instructions on how to use the *regression* feature, see Appendix A; for specific keystrokes, go to this textbook's *Online Study Center*.

TECHNOLOGY TIP Recall from Section 2.7 that when you use the *regression* feature of a graphing utility, the program may output a correlation coefficient, r. When $|r|$ is close to 1, the model is a good fit for the data.

Example 1 Finding a Least Squares Regression Line

Find the least squares regression line for $(-3, 0)$, $(-1, 1)$, $(0, 2)$, and $(2, 3)$.

Solution

Begin by constructing a table, as shown below.

x	y	xy	x^2
-3	0	0	9
-1	1	-1	1
0	2	0	0
2	3	6	4
$\sum_{i=1}^{n} x_i = -2$	$\sum_{i=1}^{n} y_i = 6$	$\sum_{i=1}^{n} x_i y_i = 5$	$\sum_{i=1}^{n} x_i^2 = 14$

Applying the system for the least squares regression line with $n = 4$ produces

$$\begin{cases} nb + \left(\sum_{i=1}^{n} x_i\right)a = \sum_{i=1}^{n} y_i \\ \left(\sum_{i=1}^{n} x_i\right)b + \left(\sum_{i=1}^{n} x_i^2\right)a = \sum_{i=1}^{n} x_i y_i \end{cases} \Rightarrow \begin{cases} 4b - 2a = 6 \\ -2b + 14a = 5 \end{cases}.$$

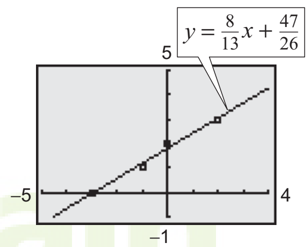


Figure B.8

Solving this system of equations produces $a = \frac{8}{13}$ and $b = \frac{47}{26}$. So, the least squares regression line is $y = \frac{8}{13}x + \frac{47}{26}$, as shown in Figure B.8.

CHECKPOINT Now try Exercise 1.

The least squares regression parabola $y = ax^2 + bx + c$ for the points

$$\{(x_1, y_1), (x_2, y_2), (x_3, y_3), \ldots, (x_n, y_n)\}$$

is obtained in a similar manner by solving the following system of three equations in three unknowns for a, b, and c.

$$\begin{cases} nc + \left(\sum_{i=1}^{n} x_i\right) b + \left(\sum_{i=1}^{n} x_i^2\right)a = \sum_{i=1}^{n} y_i \\ \left(\sum_{i=1}^{n} x_i\right)c + \left(\sum_{i=1}^{n} x_i^2\right)b + \left(\sum_{i=1}^{n} x_i^3\right)a = \sum_{i=1}^{n} x_i y_i \\ \left(\sum_{i=1}^{n} x_i^2\right)c + \left(\sum_{i=1}^{n} x_i^3\right)b + \left(\sum_{i=1}^{n} x_i^4\right)a = \sum_{i=1}^{n} x_i^2 y_i \end{cases}$$

B.2 Exercises

See www.CalcChat.com for worked-out solutions to odd-numbered exercises.

In Exercises 1–4, find the least squares regression line for the points. Verify your answer with a graphing utility.

1. $(-4, 1)$, $(-3, 3)$, $(-2, 4)$, $(-1, 6)$

2. $(0, -1)$, $(2, 0)$, $(4, 3)$, $(6, 5)$

3. $(-3, 1)$, $(-1, 2)$, $(1, 2)$, $(4, 3)$

4. $(0, -1)$, $(2, 1)$, $(3, 2)$, $(5, 3)$

Appendix C: Variation

Direct Variation

There are two basic types of linear models. The more general model has a y-intercept that is nonzero.

$$y = mx + b, \quad b \neq 0$$

The simpler model $y = kx$ has a y-intercept that is zero. In the simpler model, y is said to **vary directly** as x, or to be **directly proportional** to x.

What you should learn

- Write mathematical models for direct variation.
- Write mathematical models for direct variation as an nth power.
- Write mathematical models for inverse variation.
- Write mathematical models for joint variation.

Why you should learn it

You can use functions as models to represent a wide variety of real-life data sets. For instance, in Exercise 55 on page A42, a variation model can be used to model the water temperatures of the ocean at various depths.

Direct Variation

The following statements are equivalent.

1. y **varies directly** as x.
2. y is **directly proportional** to x.
3. $y = kx$ for some nonzero constant k.

k is the **constant of variation** or the **constant of proportionality.**

Example 1 Direct Variation

In Pennsylvania, the state income tax is directly proportional to *gross income*. You are working in Pennsylvania and your state income tax deduction is \$46.05 for a gross monthly income of \$1500. Find a mathematical model that gives the Pennsylvania state income tax in terms of gross income.

Solution

Verbal Model: State income tax $=$ k $\cdot$ Gross income

Labels:
State income tax $= y$ (dollars)
Gross income $= x$ (dollars)
Income tax rate $= k$ (percent in decimal form)

Equation: $y = kx$

To solve for k, substitute the given information in the equation $y = kx$, and then solve for k.

$$y = kx \qquad \text{Write direct variation model.}$$

$$46.05 = k(1500) \qquad \text{Substitute } y = 46.05 \text{ and } x = 1500.$$

$$0.0307 = k \qquad \text{Simplify.}$$

So, the equation (or model) for state income tax in Pennsylvania is

$$y = 0.0307x.$$

In other words, Pennsylvania has a state income tax rate of 3.07% of gross income. The graph of this equation is shown in Figure C.1.

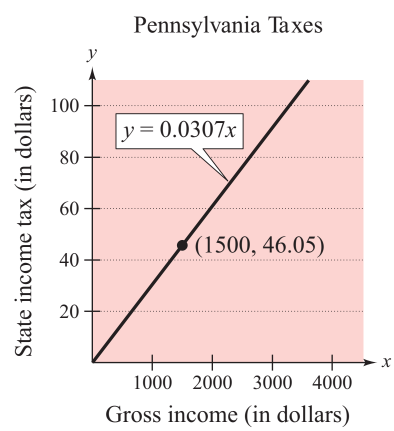


Figure C.1

CHECKPOINT Now try Exercise 7.

Direct Variation as an nth Power

Another type of direct variation relates one variable to a *power* of another variable. For example, in the formula for the area of a circle

$$A = \pi r^2$$

the area A is directly proportional to the square of the radius r. Note that for this formula, π is the constant of proportionality.

Direct Variation as an nth Power

The following statements are equivalent.

1. y **varies directly as the nth power** of x.
2. y is **directly proportional to the nth power** of x.
3. $y = kx^n$ for some constant k.

STUDY TIP

Note that the direct variation model $y = kx$ is a special case of $y = kx^n$ with $n = 1$.

Example 2 Direct Variation as an nth Power

The distance a ball rolls down an inclined plane is directly proportional to the square of the time it rolls. During the first second, the ball rolls 8 feet. (See Figure C.2.)

a. Write an equation relating the distance traveled to the time.

b. How far will the ball roll during the first 3 seconds?

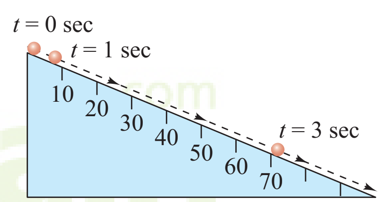


Figure C.2

Solution

a. Letting d be the distance (in feet) the ball rolls and letting t be the time (in seconds), you have

$$d = kt^2.$$

Now, because $d = 8$ when $t = 1$, you can see that $k = 8$, as follows.

$$d = kt^2$$

$$8 = k(1)^2$$

$$8 = k$$

So, the equation relating distance to time is

$$d = 8t^2.$$

b. When $t = 3$, the distance traveled is $d = 8(3)^2 = 8(9) = 72$ feet.

CHECKPOINT Now try Exercise 15.

In Examples 1 and 2, the direct variations are such that an *increase* in one variable corresponds to an *increase* in the other variable. This is also true in the model $d = \frac{1}{5}F, F > 0$, where an increase in F results in an increase in d. You should not, however, assume that this always occurs with direct variation. For example, in the model $y = -3x$, an increase in x results in a *decrease* in y, and yet y is said to vary directly as x.

Inverse Variation

> **Inverse Variation**
>
> The following statements are equivalent.
>
> **1.** y **varies inversely** as x.
>
> **2.** y is **inversely proportional** to x.
>
> **3.** $y = \dfrac{k}{x}$ for some constant k.

If x and y are related by an equation of the form $y = k/x^n$, then y varies inversely as the nth power of x (or y is inversely proportional to the nth power of x).

Some applications of variation involve problems with *both* direct and inverse variation in the same model. These types of models are said to have **combined variation.**

Example 3 Direct and Inverse Variation

A gas law states that the volume of an enclosed gas varies directly as the temperature *and* inversely as the pressure, as shown in Figure C.3. The pressure of a gas is 0.75 kilogram per square centimeter when the temperature is 294 K and the volume is 8000 cubic centimeters. (a) Write an equation relating pressure, temperature, and volume. (b) Find the pressure when the temperature is 300 K and the volume is 7000 cubic centimeters.

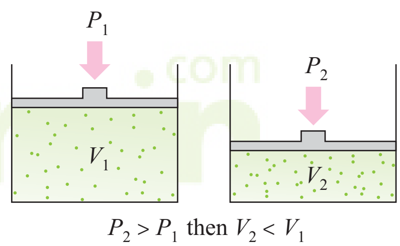


Figure C.3 *If the temperature is held constant and pressure increases, volume decreases.*

Solution

a. Let V be volume (in cubic centimeters), let P be pressure (in kilograms per square centimeter), and let T be temperature (in Kelvin). Because V varies directly as T and inversely as P, you have

$$V = \frac{kT}{P}.$$

Now, because $P = 0.75$ when $T = 294$ and $V = 8000$, you have

$$8000 = \frac{k(294)}{0.75}$$

$$k = \frac{6000}{294} = \frac{1000}{49}.$$

So, the equation relating pressure, temperature, and volume is

$$V = \frac{1000}{49}\left(\frac{T}{P}\right).$$

b. When $T = 300$ and $V = 7000$, the pressure is

$$P = \frac{1000}{49}\left(\frac{300}{7000}\right) = \frac{300}{343} \approx 0.87 \text{ kilogram per square centimeter.}$$

CHECKPOINT Now try Exercise 49.

Joint Variation

In Example 3, note that when a direct variation and an inverse variation occur in the same statement, they are coupled with the word "and." To describe two different *direct* variations in the same statement, the word **jointly** is used.

> **Joint Variation**
>
> The following statements are equivalent.
>
> **1.** z **varies jointly** as x and y.
>
> **2.** z is **jointly proportional** to x and y.
>
> **3.** $z = kxy$ for some constant k.

If x, y, and z are related by an equation of the form

$$z = kx^n y^m$$

then z varies jointly as the nth power of x and the mth power of y.

Example 4 Joint Variation

The *simple* interest for a certain savings account is jointly proportional to the time and the principal. After one quarter (3 months), the interest on a principal of \$5000 is \$43.75.

a. Write an equation relating the interest, principal, and time.

b. Find the interest after three quarters.

Solution

a. Let $I =$ interest (in dollars), $P =$ principal (in dollars), and $t =$ time (in years). Because I is jointly proportional to P and t, you have

$$I = kPt.$$

For $I = 43.75$, $P = 5000$, and $t = \frac{1}{4}$, you have

$$43.75 = k(5000)\left(\frac{1}{4}\right)$$

which implies that $k = 4(43.75)/5000 = 0.035$. So, the equation relating interest, principal, and time is

$$I = 0.035Pt$$

which is the familiar equation for simple interest where the constant of proportionality, 0.035, represents an annual interest rate of 3.5%.

b. When $P = \$5000$ and $t = \frac{3}{4}$, the interest is

$$I = (0.035)(5000)\left(\frac{3}{4}\right)$$

$$= \$131.25.$$

CHECKPOINT Now try Exercise 51.

C Exercises

See www.CalcChat.com for worked-out solutions to odd-numbered exercises.

Vocabulary Check

Fill in the blanks.

1. Direct variation models can be described as "y varies directly as x," or "y is ________ ________ to x."
2. In direct variation models of the form $y = kx$, k is called the ________ of ________.
3. The direct variation model $y = kx^n$ can be described as "y varies directly as the nth power of x," or "y is ________ ________ to the nth power of x."
4. The mathematical model $y = \frac{k}{x}$ is an example of ________ variation.
5. Mathematical models that involve both direct and inverse variation are said to have ________ variation.
6. The joint variation model $z = kxy$ can be described as "z varies jointly as x and y," or "z is ________ ________ to x and y."

In Exercises 1–4, assume that y is directly proportional to x. Use the given x-value and y-value to find a linear model that relates y and x.

1. $x = 5, \quad y = 12$
2. $x = 2, \quad y = 14$
3. $x = 10, \quad y = 2050$
4. $x = 6, \quad y = 580$

5. ***Measurement*** On a yardstick with scales in inches and centimeters, you notice that 13 inches is approximately the same length as 33 centimeters. Use this information to find a mathematical model that relates centimeters to inches. Then use the model to find the numbers of centimeters in 10 inches and 20 inches.
6. ***Measurement*** When buying gasoline, you notice that 14 gallons of gasoline is approximately the same amount of gasoline as 53 liters. Use this information to find a linear model that relates gallons to liters. Then use the model to find the numbers of liters in 5 gallons and 25 gallons.
7. ***Taxes*** Property tax is based on the assessed value of a property. A house that has an assessed value of \$150,000 has a property tax of \$5520. Find a mathematical model that gives the amount of property tax y in terms of the assessed value x of the property. Use the model to find the property tax on a house that has an assessed value of \$200,000.
8. ***Taxes*** State sales tax is based on retail price. An item that sells for \$145.99 has a sales tax of \$10.22. Find a mathematical model that gives the amount of sales tax y in terms of the retail price x. Use the model to find the sales tax on a \$540.50 purchase.

Hooke's Law **In Exercises 9 and 10, use Hooke's Law for springs, which states that the distance a spring is stretched (or compressed) varies directly as the force on the spring.**

9. A force of 265 newtons stretches a spring 0.15 meter (see figure).

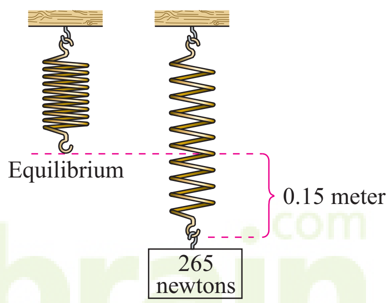


Figure for 9

(a) How far will a force of 90 newtons stretch the spring?
(b) What force is required to stretch the spring 0.1 meter?

10. A force of 220 newtons stretches a spring 0.12 meter. What force is required to stretch the spring 0.16 meter?

In Exercises 11–14, use the given value of k to complete the table for the direct variation model $y = kx^2$. Plot the points on a rectangular coordinate system.

x	2	4	6	8	10
$y = kx^2$					

11. $k = 1$
12. $k = 2$
13. $k = \frac{1}{2}$
14. $k = \frac{1}{4}$

Ecology **In Exercises 15 and 16, use the fact that the diameter of the largest particle that can be moved by a stream varies approximately directly as the square of the velocity of the stream.**

15. A stream with a velocity of $\frac{1}{4}$ mile per hour can move coarse sand particles about 0.02 inch in diameter. Approximate the velocity required to carry particles 0.12 inch in diameter.

16. A stream of velocity v can move particles of diameter d or less. By what factor does d increase when the velocity is doubled?

In Exercises 17–20, use the given value of k to complete the table for the inverse variation model $y = k/x^2$. Plot the points on a rectangular coordinate system.

x	2	4	6	8	10
$y = \dfrac{k}{x^2}$					

17. $k = 2$

18. $k = 5$

19. $k = 10$

20. $k = 20$

In Exercises 21–24, determine whether the variation model is of the form $y = kx$ or $y = k/x$, and find k.

21.

x	5	10	15	20	25
y	1	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{4}$	$\frac{1}{5}$

22.

x	5	10	15	20	25
y	2	4	6	8	10

23.

x	5	10	15	20	25
y	-3.5	-7	-10.5	-14	-17.5

24.

x	5	10	15	20	25
y	24	12	8	6	$\frac{24}{5}$

In Exercises 25–34, find a mathematical model for the verbal statement.

25. A varies directly as the square of r.

26. V varies directly as the cube of e.

27. y varies inversely as the square of x.

28. h varies inversely as the square root of s.

29. F varies directly as g and inversely as r^2.

30. z is jointly proportional to the square of x and the cube of y.

31. ***Boyle's Law:*** For a constant temperature, the pressure P of a gas is inversely proportional to the volume V of the gas.

32. ***Logistic Growth:*** The rate of growth R of a population is jointly proportional to the size S of the population and the difference between S and the maximum population size L that the environment can support.

33. ***Newton's Law of Cooling:*** The rate of change R of the temperature of an object is proportional to the difference between the temperature T of the object and the temperature T_e of the environment in which the object is placed.

34. ***Newton's Law of Universal Gravitation:*** The gravitational attraction F between two objects of masses m_1 and m_2 is proportional to the product of the masses and inversely proportional to the square of the distance r between the objects.

In Exercises 35–40, write a sentence using the variation terminology of this section to describe the formula.

35. ***Area of a triangle:*** $A = \frac{1}{2}bh$

36. ***Surface area of a sphere:*** $S = 4\pi r^2$

37. ***Volume of a sphere:*** $V = \frac{4}{3}\pi r^3$

38. ***Volume of a right circular cylinder:*** $V = \pi r^2 h$

39. ***Average speed:*** $r = \dfrac{d}{t}$

40. ***Free vibrations:*** $\omega = \sqrt{\dfrac{kg}{W}}$

In Exercises 41–48, find a mathematical model representing the statement. (In each case, determine the constant of proportionality.)

41. A varies directly as r^2. ($A = 9\pi$ when $r = 3$.)

42. y varies inversely as x. ($y = 3$ when $x = 25$.)

43. y is inversely proportional to x. ($y = 7$ when $x = 4$.)

44. z varies jointly as x and y. ($z = 64$ when $x = 4$ and $y = 8$.)

45. F is jointly proportional to r and the third power of s. ($F = 4158$ when $r = 11$ and $s = 3$.)

46. P varies directly as x and inversely as the square of y. $\left(P = \frac{28}{3}\right.$ when $x = 42$ and $\left.y = 9.\right)$

47. z varies directly as the square of x and inversely as y. ($z = 6$ when $x = 6$ and $y = 4$.)

48. v varies jointly as p and q and inversely as the square of s. ($v = 1.5$ when $p = 4.1$, $q = 6.3$, and $s = 1.2$.)

Resistance **In Exercises 49 and 50, use the fact that the resistance of a wire carrying an electrical current is directly proportional to its length and inversely proportional to its cross-sectional area.**

49. If #28 copper wire (which has a diameter of 0.0126 inch) has a resistance of 66.17 ohms per thousand feet, what length of #28 copper wire will produce a resistance of 33.5 ohms?

50. A 14-foot piece of copper wire produces a resistance of 0.05 ohm. Use the constant of proportionality from Exercise 49 to find the diameter of the wire.

51. ***Work*** The work W (in joules) done when an object is lifted varies jointly with the mass m (in kilograms) of the object and the height h (in meters) that the object is lifted. The work done when a 120-kilogram object is lifted 1.8 meters is 2116.8 joules. How much work is done when a 100-kilogram object is lifted 1.5 meters?

52. ***Spending*** The prices of three sizes of pizza at a pizza shop are as follows.

9-inch: \$8.78, 12-inch: \$11.78, 15-inch: \$14.18

You would expect that the price of a certain size of pizza would be directly proportional to its surface area. Is that the case for this pizza shop? If not, which size of pizza is the best buy?

53. ***Fluid Flow*** The velocity v of a fluid flowing in a conduit is inversely proportional to the cross-sectional area of the conduit. (Assume that the volume of the flow per unit of time is held constant.) Determine the change in the velocity of water flowing from a hose when a person places a finger over the end of the hose to decrease its cross-sectional area by 25%.

54. ***Beam Load*** The maximum load that can be safely supported by a horizontal beam varies jointly as the width of the beam and the square of its depth, and inversely as the length of the beam. Determine the changes in the maximum safe load under the following conditions.

(a) The width and length of the beam are doubled.

(b) The width and depth of the beam are doubled.

(c) All three of the dimensions are doubled.

(d) The depth of the beam is halved.

55. ***Ocean Temperatures*** An oceanographer took readings of the water temperatures C (in degrees Celsius) at several depths d (in meters). The data collected is shown in the table.

Depth, d	Temperature, C
1000	4.2°
2000	1.9°
3000	1.4°
4000	1.2°
5000	0.9°

(a) Sketch a scatter plot of the data.

(b) Does it appear that the data can be modeled by the inverse variation model $C = k/d$? If so, find k for each pair of coordinates.

(c) Determine the mean value of k from part (b) to find the inverse variation model $C = k/d$.

(d) Use a graphing utility to plot the data points and the inverse model in part (c).

(e) Use the model to approximate the depth at which the water temperature is 3°C.

56. ***Physics Experiment*** An experiment in a physics lab requires a student to measure the compressed lengths y (in centimeters) of a spring when various forces of F pounds are applied. The data is shown in the table.

Force, F	Length, y
0	0
2	1.15
4	2.3
6	3.45
8	4.6
10	5.75
12	6.9

(a) Sketch a scatter plot of the data.

(b) Does it appear that the data can be modeled by Hooke's Law? If so, estimate k. (See Exercises 9 and 10.)

(c) Use the model in part (b) to approximate the force required to compress the spring 9 centimeters.

Synthesis

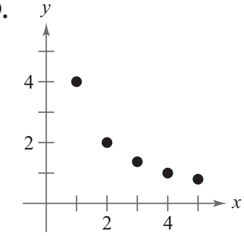
True or False? **In Exercises 57 and 58, decide whether the statement is true or false. Justify your answer.**

57. If y varies directly as x, then if x increases, y will increase as well.

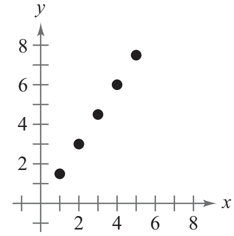
58. In the equation for kinetic energy, $E = \frac{1}{2}mv^2$, the amount of kinetic energy E is directly proportional to the mass m of an object and the square of its velocity v.

Think About It **In Exercises 59 and 60, use the graph to determine whether y varies directly as some power of x or inversely as some power of x. Explain.**

59.



60.



Appendix D: Solving Linear Equations and Inequalities

Linear Equations

A *linear equation* in one variable x is an equation that can be written in the standard form $ax + b = 0$, where a and b are real numbers with $a \neq 0$.

A linear equation in one variable, written in standard form, has exactly one solution. To see this, consider the following steps. (Remember that $a \neq 0$.)

$ax + b = 0$ Original equation

$ax = -b$ Subtract b from each side.

$x = -\frac{b}{a}$ Divide each side by a.

To solve a linear equation in x, isolate x on one side of the equation by creating a sequence of *equivalent* (and usually simpler) equations, each having the same solution(s) as the original equation. The operations that yield equivalent equations come from the Substitution Principle and the Properties of Equality studied in Chapter P.

What you should learn

- Solve linear equations in one variable.
- Solve linear inequalities in one variable.

Why you should learn it

The method of solving linear equations is used to determine the intercepts of the graph of a linear function. The method of solving linear inequalities is used to determine the domains of different functions.

Generating Equivalent Equations

An equation can be transformed into an *equivalent equation* by one or more of the following steps.

	Original Equation	*Equivalent Equation*
1. Remove symbols of grouping, combine like terms, or simplify fractions on one or both sides of the equation.	$2x - x = 4$	$x = 4$
2. Add (or subtract) the same quantity to (from) *each* side of the equation.	$x + 1 = 6$	$x = 5$
3. Multiply (or divide) *each* side of the equation by the same *nonzero* quantity.	$2x = 6$	$x = 3$
4. Interchange the two sides of the equation.	$2 = x$	$x = 2$

After solving an equation, check each solution in the original equation. For example, you can check the solution to the equation in Step 2 above as follows.

$x + 1 = 6$ Write original equation.

$5 + 1 \stackrel{?}{=} 6$ Substitute 5 for x.

$6 = 6$ Solution checks. ✓

Example 1 Solving Linear Equations

a.

$3x - 6 = 0$	Original equation
$3x - 6 + 6 = 0 + 6$	Add 6 to each side.
$3x = 6$	Simplify.
$\frac{3x}{3} = \frac{6}{3}$	Divide each side by 3.
$x = 2$	Simplify.

b.

$4(2x + 3) = 6$	Original equation
$8x + 12 = 6$	Distributive Property
$8x + 12 - 12 = 6 - 12$	Subtract 12 from each side.
$8x = -6$	Simplify.
$x = -\frac{3}{4}$	Divide each side by 8 and simplify.

CHECKPOINT Now try Exercise 15.

Linear Inequalities

Solving a linear inequality in one variable is much like solving a linear equation in one variable. To solve the inequality, you isolate the variable on one side using transformations that produce *equivalent inequalities*, which have the same solution(s) as the original inequality.

Generating Equivalent Inequalities

An inequality can be transformed into an *equivalent inequality* by one or more of the following steps.

	Original Inequality	*Equivalent Inequality*
1. Remove symbols of grouping, combine like terms, or simplify fractions on one or both sides of the inequality.	$4x + x \geq 2$	$5x \geq 2$
2. Add (or subtract) the same number to (from) *each* side of the inequality.	$x - 3 < 5$	$x < 8$
3. Multiply (or divide) each side of the inequality by the same *positive* number.	$\frac{1}{2}x > 3$	$x > 6$
4. Multiply (or divide) each side of the inequality by the same *negative* number and *reverse* the inequality symbol.	$-2x \leq 6$	$x \geq -3$

Example 2 Solving Linear Inequalities

a. $x + 5 \geq 3$ Original inequality

$x + 5 - 5 \geq 3 - 5$ Subtract 5 from each side.

$x \geq -2$ Simplify.

The solution is all real numbers greater than or equal to -2, which is denoted by $[-2, \infty)$. Check several numbers that are greater than or equal to -2 in the original inequality.

b. $-4.2m > 6.3$ Original inequality

$\frac{-4.2m}{-4.2} < \frac{6.3}{-4.2}$ Divide each side by -4.2 and reverse inequality symbol.

$m < -1.5$ Simplify.

The solution is all real numbers less than -1.5, which is denoted by $(-\infty, -1.5)$. Check several numbers that are less than -1.5 in the original inequality.

CHECKPOINT Now try Exercise 29.

STUDY TIP

Remember that when you multiply or divide by a negative number, you must *reverse the inequality symbol*, as shown in Example 2(b).

D Exercises

See www.CalcChat.com for worked-out solutions to odd-numbered exercises.

Vocabulary Check

Fill in the blanks.

1. A _______ equation in one variable x is an equation that can be written in the standard form $ax + b = 0$.
2. To solve a linear inequality, isolate the variable on one side using transformations that produce _______ .

In Exercises 1–22, solve the equation and check your solution.

1. $x + 11 = 15$
2. $x + 3 = 9$
3. $x - 2 = 5$
4. $x - 5 = 1$
5. $3x = 12$
6. $2x = 6$
7. $\frac{x}{5} = 4$
8. $\frac{x}{4} = 5$
9. $8x + 7 = 39$
10. $12x - 5 = 43$
11. $24 - 7x = 3$
12. $13 + 6x = 61$
13. $8x - 5 = 3x + 20$
14. $7x + 3 = 3x - 17$
15. $-2(x + 5) = 10$
16. $4(3 - x) = 9$
17. $2x + 3 = 2x - 2$
18. $8(x - 2) = 4(2x - 4)$
19. $\frac{3}{2}(x + 5) - \frac{1}{4}(x + 24) = 0$
20. $\frac{3}{2}x + \frac{1}{4}(x - 2) = 10$
21. $0.25x + 0.75(10 - x) = 3$
22. $0.60x + 0.40(100 - x) = 50$

In Exercises 23–44, solve the inequality and check your solution.

23. $x + 6 < 8$
24. $3 + x > -10$
25. $-x - 8 > -17$
26. $-3 + x < 19$
27. $6 + x \leq -8$
28. $x - 10 \geq -6$
29. $\frac{4}{5}x > 8$
30. $\frac{2}{3}x < -4$
31. $-\frac{3}{4}x > -3$
32. $-\frac{1}{6}x < -2$
33. $4x < 12$
34. $10x > -40$
35. $-11x \leq -22$
36. $-7x \geq 21$
37. $x - 3(x + 1) \geq 7$
38. $2(4x - 5) - 3x \leq -15$
39. $7x - 12 < 4x + 6$
40. $11 - 6x \leq 2x + 7$
41. $\frac{3}{4}x - 6 \leq x - 7$
42. $3 + \frac{2}{7}x > x - 2$
43. $3.6x + 11 \geq -3.4$
44. $15.6 - 1.3x < -5.2$

Appendix E: Systems of Inequalities

E.1 Solving Systems of Inequalities

What you should learn

- Sketch graphs of inequalities in two variables.
- Solve systems of inequalities.
- Use systems of inequalities in two variables to model and solve real-life problems.

Why you should learn it

Systems of inequalities in two variables can be used to model and solve real-life problems. For instance, Exercise 81 on page A55 shows how to use a system of inequalities to analyze the compositions of dietary supplements.

The Graph of an Inequality

The statements $3x - 2y < 6$ and $2x^2 + 3y^2 \geq 6$ are inequalities in two variables. An ordered pair (a, b) is a **solution of an inequality** in x and y if the inequality is true when a and b are substituted for x and y, respectively. The **graph of an inequality** is the collection of all solutions of the inequality. To sketch the graph of an inequality, begin by sketching the graph of the *corresponding equation*. The graph of the equation will normally separate the plane into two or more regions. In each such region, one of the following must be true.

1. *All* points in the region are solutions of the inequality.
2. *No* point in the region is a solution of the inequality.

So, you can determine whether the points in an entire region satisfy the inequality by simply testing *one* point in the region.

Sketching the Graph of an Inequality in Two Variables

1. Replace the inequality sign with an equal sign and sketch the graph of the corresponding equation. Use a dashed line for $<$ or $>$ and a solid line for $\leq$ or $\geq$. (A dashed line means that all points on the line or curve *are not* solutions of the inequality. A solid line means that all points on the line or curve *are* solutions of the inequality.)
2. Test one point in each of the regions formed by the graph in Step 1. If the point satisfies the inequality, shade the entire region to denote that every point in the region satisfies the inequality.

Example 1 Sketching the Graph of an Inequality

Sketch the graph of $y \geq x^2 - 1$ by hand.

Solution

Begin by graphing the corresponding *equation* $y = x^2 - 1$, which is a parabola, as shown in Figure E.1. By testing a point *above* the parabola $(0, 0)$ and a point *below* the parabola $(0, -2)$, you can see that $(0, 0)$ satisfies the inequality because $0 \geq 0^2 - 1$ and that $(0, -2)$ does not satisfy the inequality because $-2 \ngeq 0^2 - 1$. So, the points that satisfy the inequality are those lying above and those lying on the parabola.

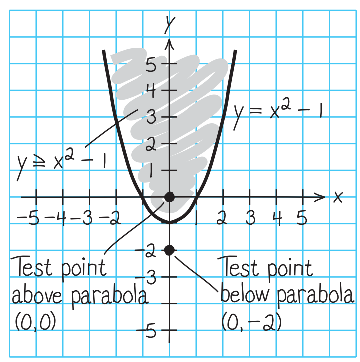


Figure E.1

CHECKPOINT Now try Exercise 9.

The inequality in Example 1 is a nonlinear inequality in two variables. Most of the following examples involve **linear inequalities** such as $ax + by < c$ (a and b are not both zero). The graph of a linear inequality is a half-plane lying on one side of the line $ax + by = c$.

Example 2 Sketching the Graphs of Linear Inequalities

Sketch the graph of each linear inequality.

a. $x > -2$ **b.** $y \leq 3$

Solution

a. The graph of the corresponding equation $x = -2$ is a vertical line. The points that satisfy the inequality $x > -2$ are those lying to the right of (but not on) this line, as shown in Figure E.2.

b. The graph of the corresponding equation $y = 3$ is a horizontal line. The points that satisfy the inequality $y \leq 3$ are those lying below (or on) this line, as shown in Figure E.3.

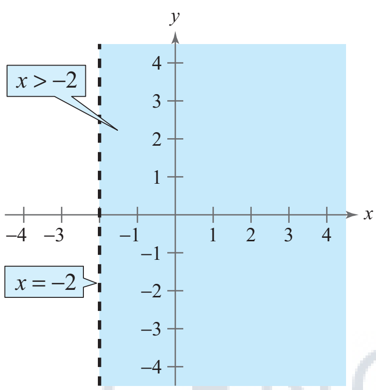


Figure E.2

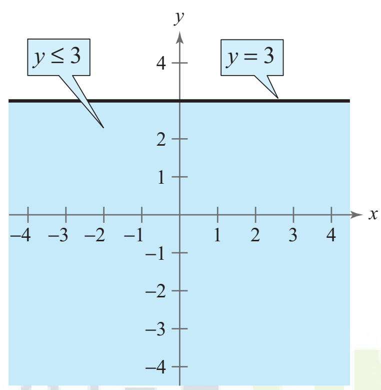


Figure E.3

CHECKPOINT Now try Exercise 15.

Example 3 Sketching the Graph of a Linear Inequality

Sketch the graph of $x - y < 2$.

Solution

The graph of the corresponding equation $x - y = 2$ is a line, as shown in Figure E.4. Because the origin $(0, 0)$ satisfies the inequality, the graph consists of the half-plane lying above the line. (Try checking a point below the line. Regardless of which point below the line you choose, you will see that it does not satisfy the inequality.)

CHECKPOINT Now try Exercise 17.

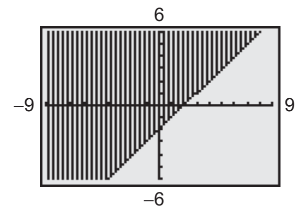
To graph a linear inequality, it can help to write the inequality in slope-intercept form. For instance, by writing $x - y < 2$ in Example 3 in the form

$$y > x - 2$$

you can see that the solution points lie *above* the line $y = x - 2$ (or $x - y = 2$), as shown in Figure E.4.

TECHNOLOGY TIP

A graphing utility can be used to graph an inequality. For instance, to graph $y \geq x - 2$, enter $y = x - 2$ and use the *shade* feature of the graphing utility to shade the correct part of the graph. You should obtain the graph shown below.



For instructions on how to use the *shade* feature, see Appendix A; for specific keystrokes, go to this textbook's *Online Study Center*.

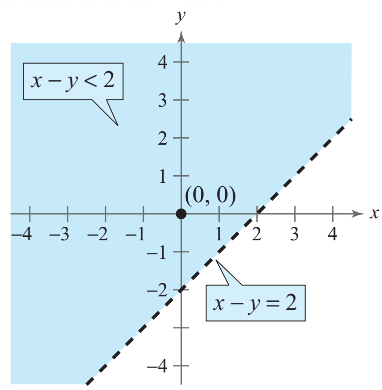


Figure E.4

Systems of Inequalities

Many practical problems in business, science, and engineering involve systems of linear inequalities. A **solution of a system of inequalities** in x and y is a point (x, y) that satisfies each inequality in the system.

To sketch the graph of a system of inequalities in two variables, first sketch the graph of each individual inequality (on the same coordinate system) and then find the region that is *common* to every graph in the system. For systems of *linear* inequalities, it is helpful to find the vertices of the solution region.

Example 4 Solving a System of Inequalities

Sketch the graph (and label the vertices) of the solution set of the system.

$$\begin{cases} x - y < 2 & \text{Inequality 1} \\ x > -2 & \text{Inequality 2} \\ y \leq 3 & \text{Inequality 3} \end{cases}$$

Solution

The graphs of these inequalities are shown in Figures E.4, E.2, and E.3, respectively. The triangular region common to all three graphs can be found by superimposing the graphs on the same coordinate system, as shown in Figure E.5. To find the vertices of the region, solve the three systems of corresponding equations obtained by taking pairs of equations representing the boundaries of the individual regions and solving these pairs of equations.

Vertex A: $(-2, -4)$ $\begin{cases} x - y = 2 \\ x = -2 \end{cases}$

Vertex B: $(5, 3)$ $\begin{cases} x - y = 2 \\ y = 3 \end{cases}$

Vertex C: $(-2, 3)$ $\begin{cases} x = -2 \\ y = 3 \end{cases}$

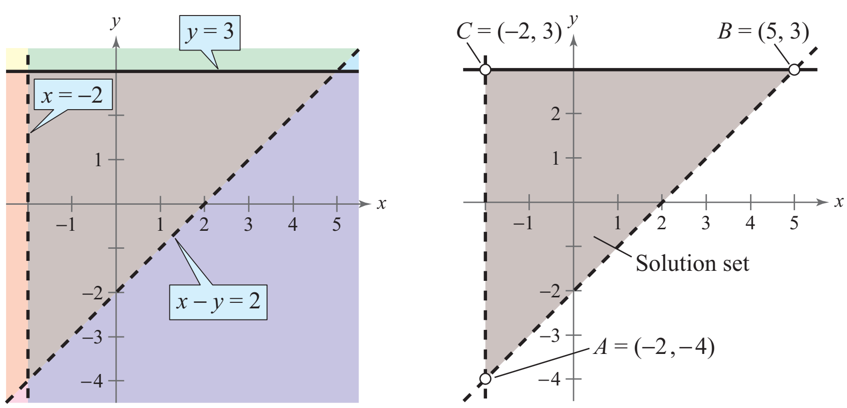


Figure E.5

Note in Figure E.5 that the vertices of the region are represented by open dots. This means that the vertices *are not* solutions of the system of inequalities.

CHECKPOINT Now try Exercise 47.

STUDY TIP

Using different colored pencils to shade the solution of each inequality in a system makes identifying the solution of the system of inequalities easier. The region common to every graph in the system is where all shaded regions overlap. This region represents the solution set of the system.

For the triangular region shown in Figure E.5, each point of intersection of a pair of boundary lines corresponds to a vertex. With more complicated regions, two border lines can sometimes intersect at a point that is not a vertex of the region, as shown in Figure E.6. To keep track of which points of intersection are actually vertices of the region, you should sketch the region and refer to your sketch as you find each point of intersection.

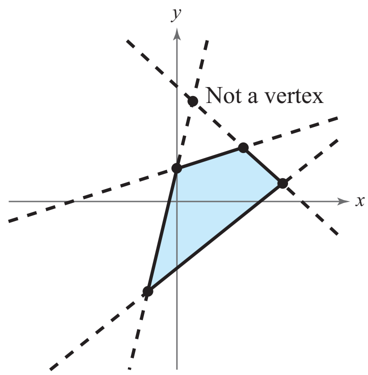


Figure E.6

Example 5 Solving a System of Inequalities

Sketch the region containing all points that satisfy the system of inequalities.

$$\begin{cases} x^2 - y \le 1 & \text{Inequality 1} \\ -x + y \le 1 & \text{Inequality 2} \end{cases}$$

Solution

As shown in Figure E.7, the points that satisfy the inequality $x^2 - y \le 1$ are the points lying above (or on) the parabola given by

$$y = x^2 - 1. \qquad \text{Parabola}$$

The points that satisfy the inequality $-x + y \le 1$ are the points lying below (or on) the line given by

$$y = x + 1. \qquad \text{Line}$$

To find the points of intersection of the parabola and the line, solve the system of corresponding equations.

$$\begin{cases} x^2 - y = 1 \\ -x + y = 1 \end{cases}$$

Using the method of substitution, you can find the solutions to be $(-1, 0)$ and $(2, 3)$. So, the region containing all points that satisfy the system is indicated by the purple shaded region in Figure E.7.

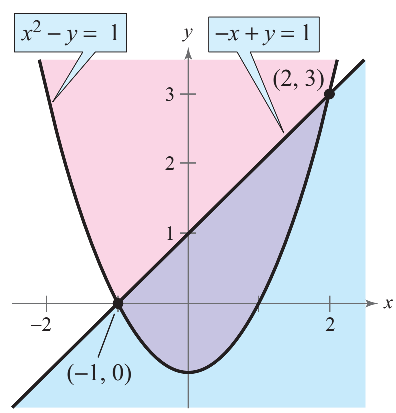


Figure E.7

CHECKPOINT Now try Exercise 55.

When solving a system of inequalities, you should be aware that the system might have no solution, or it might be represented by an unbounded region in the plane. These two possibilities are shown in Examples 6 and 7.

Example 6 A System with No Solution

Sketch the solution set of the system of inequalities.

$$\begin{cases} x + y > 3 & \text{Inequality 1} \\ x + y < -1 & \text{Inequality 2} \end{cases}$$

Solution

From the way the system is written, it is clear that the system has no solution, because the quantity $(x + y)$ cannot be both less than -1 and greater than 3. Graphically, the inequality $x + y > 3$ is represented by the half-plane lying above the line $x + y = 3$, and the inequality $x + y < -1$ is represented by the half-plane lying below the line $x + y = -1$, as shown in Figure E.8. These two half-planes have no points in common. So the system of inequalities has no solution.

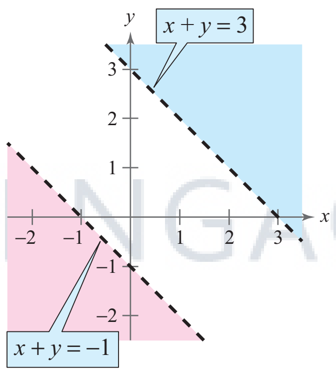


Figure E.8 ***No Solution***

CHECKPOINT Now try Exercise 51.

> **STUDY TIP**
>
> Remember that a solid line represents points on the boundary of a region that are solutions to the system of inequalities and a dashed line represents points on the boundary of a region that are not solutions. An unbounded region of a graph extending infinitely in the plane should not be bounded by a solid or dashed line, as shown in Figure E.9.

Example 7 An Unbounded Solution Set

Sketch the solution set of the system of inequalities.

$$\begin{cases} x + y < 3 & \text{Inequality 1} \\ x + 2y > 3 & \text{Inequality 2} \end{cases}$$

Solution

The graph of the inequality $x + y < 3$ is the half-plane that lies below the line $x + y = 3$, as shown in Figure E.9. The graph of the inequality $x + 2y > 3$ is the half-plane that lies above the line $x + 2y = 3$. The intersection of these two half-planes is an *infinite wedge* that has a vertex at $(3, 0)$. This unbounded region represents the solution set.

CHECKPOINT Now try Exercise 53.

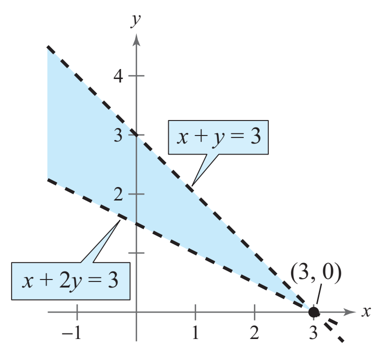


Figure E.9 ***Unbounded Region***

Applications

The next example discusses two concepts that economists call *consumer surplus* and *producer surplus*. As shown in Figure E.10, the *point of equilibrium* is defined by the price p and the number of units x that satisfy both the demand and supply equations. Consumer surplus is defined as the area of the region that lies *below* the demand curve, *above* the horizontal line passing through the equilibrium point, and to the right of the p-axis. Similarly, the producer surplus is defined as the area of the region that lies *above* the supply curve, *below* the horizontal line passing through the equilibrium point, and to the right of the p-axis. The consumer surplus is a measure of the amount that consumers would have been willing to pay *above what they actually paid*, whereas the producer surplus is a measure of the amount that producers would have been willing to receive *below what they actually received*.

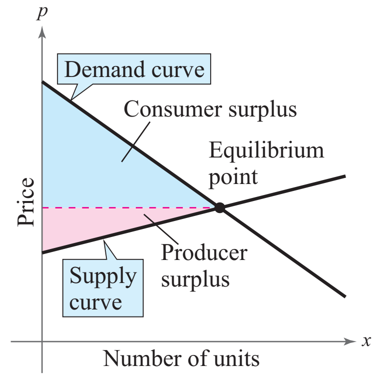


Figure E.10

Example 8 Consumer Surplus and Producer Surplus

The demand and supply functions for a new type of calculator are given by

$$\begin{cases} p = 150 - 0.00001x & \text{Demand equation} \\ p = 60 + 0.00002x & \text{Supply equation} \end{cases}$$

where p is the price (in dollars) and x represents the number of units. Find the consumer surplus and producer surplus for these two equations.

Solution

Begin by finding the point of equilibrium by setting the two equations equal to each other and solving for x.

$$60 + 0.00002x = 150 - 0.00001x \qquad \text{Set equations equal to each other.}$$

$$0.00003x = 90 \qquad \text{Combine like terms.}$$

$$x = 3{,}000{,}000 \qquad \text{Solve for } x.$$

So, the solution is $x = 3{,}000{,}000$, which corresponds to an equilibrium price of $p = \$120$. So, the consumer surplus and producer surplus are the areas of the following triangular regions.

Consumer Surplus

$$\begin{cases} p \le 150 - 0.00001x \\ p \ge 120 \\ x \ge 0 \end{cases}$$

Producer Surplus

$$\begin{cases} p \ge 60 + 0.00002x \\ p \le 120 \\ x \ge 0 \end{cases}$$

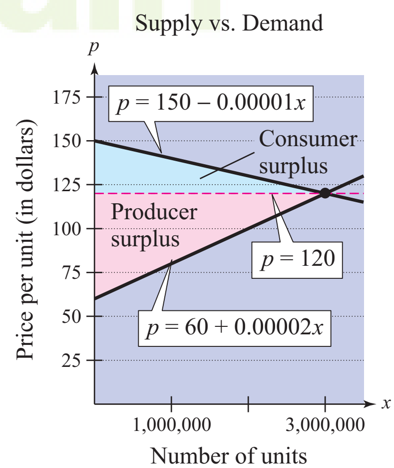


Figure E.11

In Figure E.11, you can see that the consumer and producer surpluses are defined as the areas of the shaded triangles.

$$\text{Consumer surplus} = \tfrac{1}{2}(\text{base})(\text{height}) = \tfrac{1}{2}(3{,}000{,}000)(30) = \$45{,}000{,}000$$

$$\text{Producer surplus} = \tfrac{1}{2}(\text{base})(\text{height}) = \tfrac{1}{2}(3{,}000{,}000)(60) = \$90{,}000{,}000$$

CHECKPOINT Now try Exercise 75.

Example 9 Nutrition

The minimum daily requirements from the liquid portion of a diet are 300 calories, 36 units of vitamin A, and 90 units of vitamin C. A cup of dietary drink X provides 60 calories, 12 units of vitamin A, and 10 units of vitamin C. A cup of dietary drink Y provides 60 calories, 6 units of vitamin A, and 30 units of vitamin C. Set up a system of linear inequalities that describes how many cups of each drink should be consumed each day to meet the minimum daily requirements for calories and vitamins.

Solution

Begin by letting x and y represent the following.

$x =$ number of cups of dietary drink X

$y =$ number of cups of dietary drink Y

To meet the minimum daily requirements, the following inequalities must be satisfied.

$$\begin{cases} 60x + 60y \geq 300 & \text{Calories} \\ 12x + 6y \geq 36 & \text{Vitamin A} \\ 10x + 30y \geq 90 & \text{Vitamin C} \\ x \geq 0 & \\ y \geq 0 & \end{cases}$$

The last two inequalities are included because x and y cannot be negative. The graph of this system of inequalities is shown in Figure E.12. (More is said about this application in Example 6 in Section E.2.)

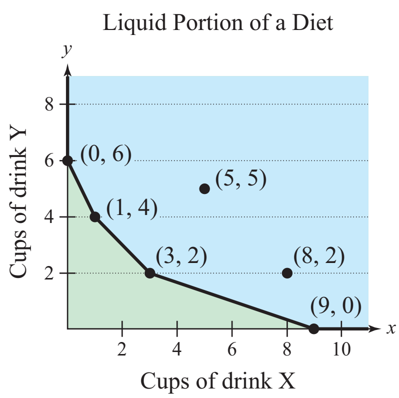


Figure E.12

From the graph, you can see that two solutions (other than the vertices) that will meet the minimum daily requirements for calories and vitamins are (5, 5) and (8, 2). There are many other solutions.

CHECKPOINT Now try Exercise 81.

STUDY TIP

When using a system of inequalities to represent a real-life application in which the variables cannot be negative, remember to include inequalities for this constraint. For instance, in Example 9, x and y cannot be negative, so the inequalities $x \geq 0$ and $y \geq 0$ must be included in the system.

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E.1 Exercises

See www.CalcChat.com for worked-out solutions to odd-numbered exercises.

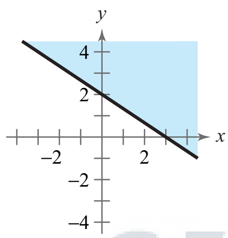
Vocabulary Check

Fill in the blanks.

1. An ordered pair (a, b) is a _______ of an inequality in x and y if the inequality is true when a and b are substituted for x and y, respectively.
2. The _______ of an inequality is the collection of all solutions of the inequality.
3. The graph of a _______ inequality is a half-plane lying on one side of the line $ax + by = c$.
4. The _______ of _______ is defined by the price p and the number of units x that satisfy both the demand and supply equations.

In Exercises 1–8, match the inequality with its graph. [The graphs are labeled (a), (b), (c), (d), (e), (f), (g), and (h).]

(a)



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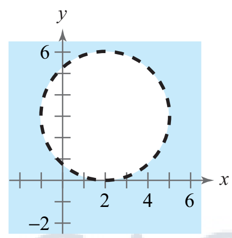

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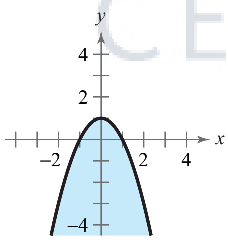

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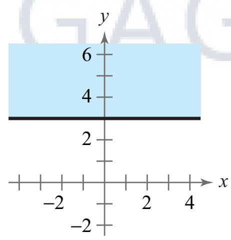
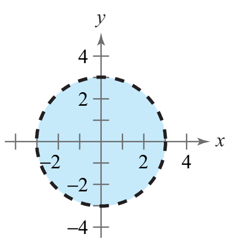


(e)



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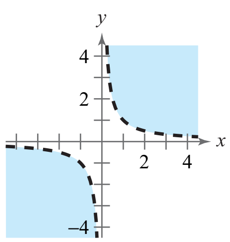

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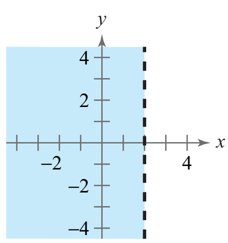
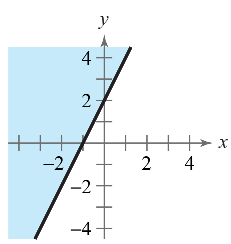


(h)



1. $x < 2$
2. $y \geq 3$
3. $2x + 3y \geq 6$
4. $2x - y \leq -2$
5. $x^2 + y^2 < 9$
6. $(x - 2)^2 + (y - 3)^2 > 9$
7. $xy > 1$
8. $y \leq 1 - x^2$

In Exercises 9–28, sketch the graph of the inequality.

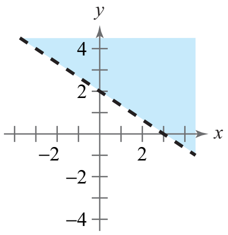
9. $y < 2 - x^2$
10. $y - 4 \leq x^2$
11. $y^2 + 1 \geq x$
12. $y^2 - x < 0$
13. $x \geq 4$
14. $x \leq -5$
15. $y \geq -1$
16. $y \leq 3$
17. $2y - x \geq 4$
18. $5x + 3y \geq -15$
19. $2x + 3y < 6$
20. $5x - 2y > 10$
21. $4x - 3y \leq 24$
22. $2x + 7y \leq 28$
23. $y > 3x^2 + 1$
24. $y + 9 \geq x^2$
25. $2x - y^2 > 0$
26. $4x + y^2 > 1$
27. $(x + 1)^2 + y^2 < 9$
28. $(x - 1)^2 + (y - 4)^2 > 9$

In Exercises 29–40, use a graphing utility to graph the inequality. Use the *shade* feature to shade the region representing the solution.

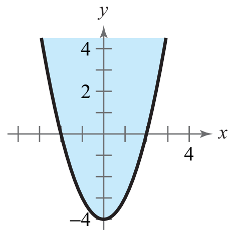
29. $y \geq \frac{2}{3}x - 1$
30. $y \leq 6 - \frac{3}{2}x$
31. $y < -3.8x + 1.1$
32. $y \geq -20.74 + 2.66x$
33. $x^2 + 5y - 10 \leq 0$
34. $2x^2 - y - 3 > 0$
35. $y \leq \dfrac{1}{1 + x^2}$
36. $y > \dfrac{-10}{x^2 + x + 4}$
37. $y < \ln x$
38. $y \geq 4 - \ln(x + 5)$
39. $y > 3^{-x-4}$
40. $y \leq 2^{2x-1} - 3$

In Exercises 41–44, write an inequality for the shaded region shown in the graph.

41.



42.



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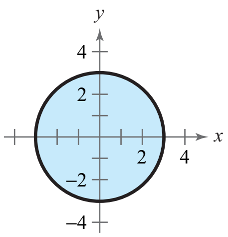

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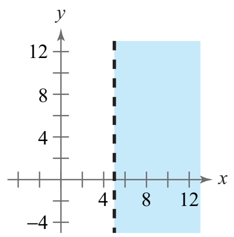


In Exercises 45 and 46, determine whether each ordered pair is a solution of the system of inequalities.

45. $\begin{cases} -2x + 5y \geq 3 \\ y < 4 \\ -4x + 2y < 7 \end{cases}$ (a) $(0, 2)$ (b) $(-6, 4)$ (c) $(-8, -2)$ (d) $(-3, 2)$

46. $\begin{cases} x^2 + y^2 \geq 36 \\ -3x + y \leq 10 \\ \frac{2}{3}x - y \geq 5 \end{cases}$ (a) $(-1, 7)$ (b) $(-5, 1)$ (c) $(6, 0)$ (d) $(4, -8)$

In Exercises 47–64, sketch the graph of the solution of the system of inequalities.

47. $\begin{cases} x + y \leq 1 \\ -x + y \leq 1 \\ y \geq 0 \end{cases}$

48. $\begin{cases} 3x + 2y < 6 \\ x > 0 \\ y > 0 \end{cases}$

49. $\begin{cases} -3x + 2y < 6 \\ x - 4y > -2 \\ 2x + y < 3 \end{cases}$

50. $\begin{cases} x - 7y > -36 \\ 5x + 2y > 5 \\ 6x - 5y > 6 \end{cases}$

51. $\begin{cases} 3x + y \leq y^2 \\ x - y > 0 \end{cases}$

52. $\begin{cases} y^2 - 3x \geq 9 \\ x + y \geq -3 \end{cases}$

53. $\begin{cases} 2x + y < 2 \\ x + 3y > 2 \end{cases}$

54. $\begin{cases} x - 2y < -6 \\ 2x - 4y > -9 \end{cases}$

55. $\begin{cases} x < y^2 \\ x > y + 2 \end{cases}$

56. $\begin{cases} x - y^2 > 0 \\ x - y < 2 \end{cases}$

57. $\begin{cases} x^2 + y^2 \leq 9 \\ x^2 + y^2 \geq 1 \end{cases}$

58. $\begin{cases} x^2 + y^2 \leq 25 \\ 4x - 3y \leq 0 \end{cases}$

59. $\begin{cases} y \leq \sqrt{3x} + 1 \\ y \geq x^2 + 1 \end{cases}$

60. $\begin{cases} y < -x^2 + 2x + 3 \\ y > x^2 - 4x + 3 \end{cases}$

61. $\begin{cases} y < x^3 - 2x + 1 \\ y > -2x \\ x \leq 1 \end{cases}$

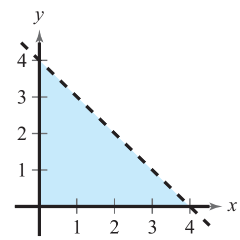
62. $\begin{cases} y \geq x^4 - 2x^2 + 1 \\ y \leq 1 - x^2 \end{cases}$

63. $\begin{cases} x^2 y \geq 1 \\ 0 < x \leq 4 \\ y \leq 4 \end{cases}$

64. $\begin{cases} y \leq e^{-x^2/2} \\ y \geq 0 \\ -2 \leq x \leq 2 \end{cases}$

In Exercises 65–74, find a set of inequalities to describe the region.

65.



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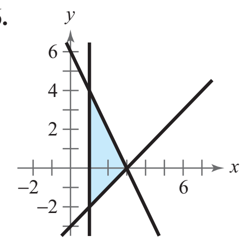
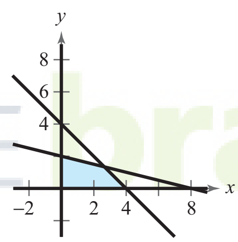
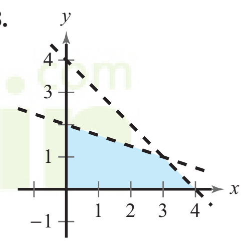


67.



68.



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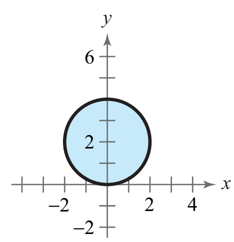
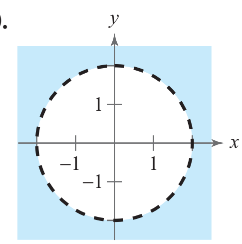


70.



71. Rectangle: Vertices at $(2, 1)$, $(5, 1)$, $(5, 7)$, $(2, 7)$

72. Parallelogram: Vertices at $(0, 0)$, $(4, 0)$, $(1, 4)$, $(5, 4)$

73. Triangle: Vertices at $(0, 0)$, $(5, 0)$, $(2, 3)$

74. Triangle: Vertices at $(-1, 0)$, $(1, 0)$, $(0, 1)$

Supply and Demand **In Exercises 75–78, (a) graph the system representing the consumer surplus and producer surplus for the supply and demand equations, and shade the region representing the solution of the system, and (b) find the consumer surplus and the producer surplus.**

	Demand	*Supply*
75.	$p = 50 - 0.5x$	$p = 0.125x$

	Demand	*Supply*
76.	$p = 100 - 0.05x$	$p = 25 + 0.1x$
77.	$p = 300 - 0.0002x$	$p = 225 + 0.0005x$
78.	$p = 140 - 0.00002x$	$p = 80 + 0.00001x$

In Exercises 79–82, (a) find a system of inequalities that models the problem and (b) graph the system, shading the region that represents the solution of the system.

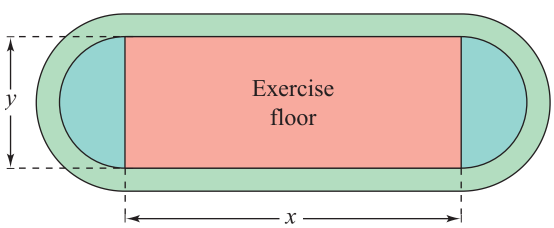
79. ***Investment Analysis*** A person plans to invest some or all of \$30,000 in two different interest-bearing accounts. Each account is to contain at least \$7500, and one account should have at least twice the amount that is in the other account.

80. ***Ticket Sales*** For a summer concert event, one type of ticket costs \$20 and another costs \$35. The promoter of the concert must sell at least 20,000 tickets, including at least 10,000 of the \$20 tickets and at least 5000 of the \$35 tickets, and the gross receipts must total at least \$300,000 in order for the concert to be held.

81. ***Nutrition*** A dietitian is asked to design a special dietary supplement using two different foods. The minimum daily requirements of the new supplement are 280 units of calcium, 160 units of iron, and 180 units of vitamin B. Each ounce of food X contains 20 units of calcium, 15 units of iron, and 10 units of vitamin B. Each ounce of food Y contains 10 units of calcium, 10 units of iron, and 20 units of vitamin B.

82. ***Inventory*** A store sells two models of computers. Because of the demand, the store stocks at least twice as many units of model A as units of model B. The costs to the store for models A and B are \$800 and \$1200, respectively. The management does not want more than \$20,000 in computer inventory at any one time, and it wants at least four model A computers and two model B computers in inventory at all times.

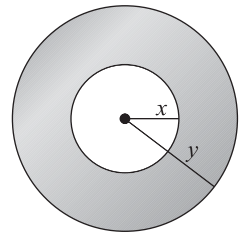
83. ***Construction*** You design an exercise facility that has an indoor running track with an exercise floor inside the track (see figure). The track must be at least 125 meters long, and the exercise floor must have an area of at least 500 square meters.



(a) Find a system of inequalities describing the requirements of the facility.

(b) Sketch the graph of the system in part (a).

84. ***Graphical Reasoning*** Two concentric circles have radii of x and y meters, where $y > x$ (see figure). The area between the boundaries of the circles must be at least 10 square meters.



(a) Find a system of inequalities describing the constraints on the circles.

(b) Graph the inequality in part (a).

(c) Identify the graph of the line $y = x$ in relation to the boundary of the inequality. Explain its meaning in the context of the problem.

Synthesis

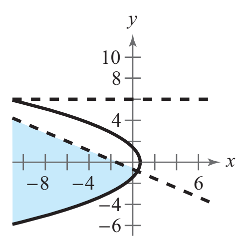
True or False? **In Exercises 85 and 86, determine whether the statement is true or false. Justify your answer.**

85. The area of the figure defined by the system below is 99 square units.

$$\begin{cases} x \geq -3 \\ x \leq 6 \\ y \leq 5 \\ y \geq -6 \end{cases}$$

86. The graph below shows the solution of the system

$$\begin{cases} y \leq 6 \\ -4x - 9y > 6. \\ 3x + y^2 \geq 2 \end{cases}$$



87. ***Think About It*** After graphing the boundary of an inequality in x and y, how do you decide on which side of the boundary the solution set of the inequality lies?

88. ***Writing*** Describe the difference between the solution set of a system of equations and the solution set of a system of inequalities.

E.2 Linear Programming

What you should learn

- Solve linear programming problems.
- Use linear programming to model and solve real-life problems.

Why you should learn it

Linear programming is a powerful tool used in business and industry to manage resources effectively in order to maximize profits or minimize costs. For instance, Exercise 36 on page A64 shows how to use linear programming to analyze the profitability of two models of snowboards.

Linear Programming: A Graphical Approach

Many applications in business and economics involve a process called **optimization,** in which you are asked to find the minimum or maximum value of a quantity. In this section, you will study an optimization strategy called **linear programming.**

A two-dimensional linear programming problem consists of a linear **objective function** and a system of linear inequalities called **constraints.** The objective function gives the quantity that is to be maximized (or minimized), and the constraints determine the set of **feasible solutions.** For example, suppose you are asked to maximize the value of

$z = ax + by$ Objective function

subject to a set of constraints that determines the region in Figure E.13. Because every point in the shaded region satisfies each constraint, it is not clear how you should find the point that yields a maximum value of z. Fortunately, it can be shown that if there is an optimal solution, it must occur at one of the vertices. So, *you can find the maximum value of z by testing z at each of the vertices.*

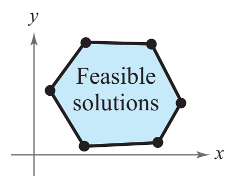


Figure E.13

Optimal Solution of a Linear Programming Problem

If a linear programming problem has a solution, it must occur at a vertex of the set of feasible solutions. If there is more than one solution, at least one of them must occur at such a vertex. In either case, the value of the objective function is unique.

Here are some guidelines for solving a linear programming problem in two variables in which an objective function is to be maximized or minimized.

Solving a Linear Programming Problem

1. Sketch the region corresponding to the system of constraints. (The points inside or on the boundary of the region are *feasible solutions*.)
2. Find the vertices of the region.
3. Test the objective function at each of the vertices and select the values of the variables that optimize the objective function. For a bounded region, both a minimum and a maximum value will exist. (For an unbounded region, *if* an optimal solution exists, it will occur at a vertex.)

Example 1 Solving a Linear Programming Problem

Find the maximum value of

$$z = 3x + 2y \qquad \text{Objective function}$$

subject to the following constraints.

$$\left.\begin{aligned} x &\geq 0 \\ y &\geq 0 \\ x + 2y &\leq 4 \\ x - y &\leq 1 \end{aligned}\right\} \qquad \text{Constraints}$$

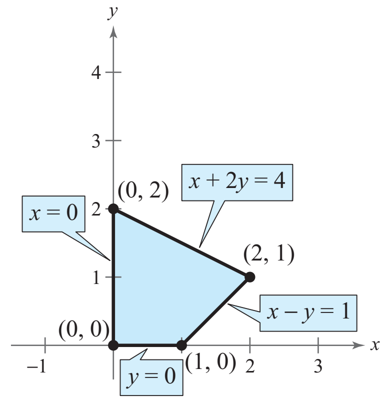


Figure E.14

Solution

The constraints form the region shown in Figure E.14. At the four vertices of this region, the objective function has the following values.

At (0, 0): $z = 3(0) + 2(0) = 0$

At (1, 0): $z = 3(1) + 2(0) = 3$

At (2, 1): $z = 3(2) + 2(1) = 8$ Maximum value of z

At (0, 2): $z = 3(0) + 2(2) = 4$

So, the maximum value of z is 8, and this value occurs when $x = 2$ and $y = 1$.

CHECKPOINT Now try Exercise 13.

In Example 1, try testing some of the *interior* points in the region. You will see that the corresponding values of z are less than 8. Here are some examples.

At (1, 1): $z = 3(1) + 2(1) = 5$

At $\left(1, \frac{1}{2}\right)$: $z = 3(1) + 2\left(\frac{1}{2}\right) = 4$

At $\left(\frac{1}{2}, \frac{3}{2}\right)$: $z = 3\left(\frac{1}{2}\right) + 2\left(\frac{3}{2}\right) = \frac{9}{2}$

STUDY TIP

Remember that a vertex of a region can be found using a system of linear equations. The system will consist of the equations of the lines passing through the vertex.

To see why the maximum value of the objective function in Example 1 must occur at a vertex, consider writing the objective function in the form

$$y = -\frac{3}{2}x + \frac{z}{2} \qquad \text{Family of lines}$$

where $z/2$ is the y-intercept of the objective function. This equation represents a family of lines, each of slope $-\frac{3}{2}$. Of these infinitely many lines, you want the one that has the largest z-value while still intersecting the region determined by the constraints. In other words, of all the lines with a slope of $-\frac{3}{2}$, you want the one that has the largest y-intercept *and* intersects the given region, as shown in Figure E.15. It should be clear that such a line will pass through one (or more) of the vertices of the region.

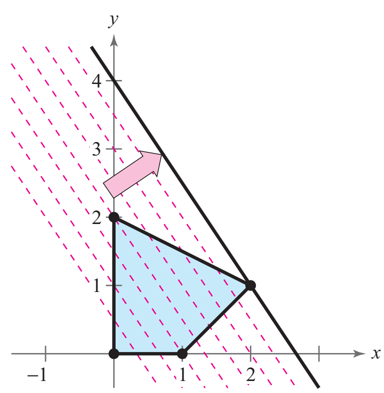


Figure E.15

The next example shows that the same basic procedure can be used to solve a problem in which the objective function is to be *minimized*.

Example 2 Solving a Linear Programming Problem

Find the minimum value of

$$z = 5x + 7y \qquad \text{Objective function}$$

where $x \geq 0$ and $y \geq 0$, subject to the following constraints.

$$\left.\begin{aligned} 2x + 3y &\geq 6 \\ 3x - y &\leq 15 \\ -x + y &\leq 4 \\ 2x + 5y &\leq 27 \end{aligned}\right\} \quad \text{Constraints}$$

Solution

The region bounded by the constraints is shown in Figure E.16. By testing the objective function at each vertex, you obtain the following.

At $(0, 2)$: $z = 5(0) + 7(2) = 14$ Minimum value of z

At $(0, 4)$: $z = 5(0) + 7(4) = 28$

At $(1, 5)$: $z = 5(1) + 7(5) = 40$

At $(6, 3)$: $z = 5(6) + 7(3) = 51$

At $(5, 0)$: $z = 5(5) + 7(0) = 25$

At $(3, 0)$: $z = 5(3) + 7(0) = 15$

So, the minimum value of z is 14, and this value occurs when $x = 0$ and $y = 2$.

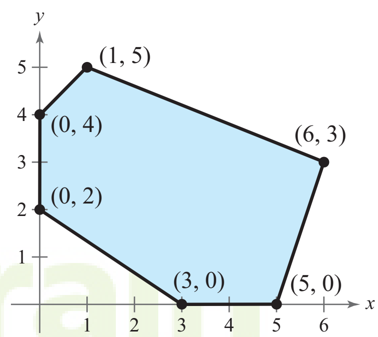


Figure E.16

CHECKPOINT Now try Exercise 15.

Example 3 Solving a Linear Programming Problem

Find the maximum value of

$$z = 5x + 7y \qquad \text{Objective function}$$

where $x \geq 0$ and $y \geq 0$, subject to the following constraints.

$$\left.\begin{aligned} 2x + 3y &\geq 6 \\ 3x - y &\leq 15 \\ -x + y &\leq 4 \\ 2x + 5y &\leq 27 \end{aligned}\right\} \quad \text{Constraints}$$

Solution

This linear programming problem is identical to that given in Example 2 above, *except* that the objective function is *maximized* instead of minimized. Using the values of z at the vertices shown in Example 2, you can conclude that the maximum value of z is 51, and that this value occurs when $x = 6$ and $y = 3$.

CHECKPOINT Now try Exercise 21.

It is possible for the maximum (or minimum) value in a linear programming problem to occur at *two* different vertices. For instance, at the vertices of the region shown in Figure E.17, the objective function

$z = 2x + 2y$ Objective function

has the following values.

At (0, 0): $z = 2(0) + 2(0) = 0$

At (0, 4): $z = 2(0) + 2(4) = 8$

At (2, 4): $z = 2(2) + 2(4) = 12$ Maximum value of z

At (5, 1): $z = 2(5) + 2(1) = 12$ Maximum value of z

At (5, 0): $z = 2(5) + 2(0) = 10$

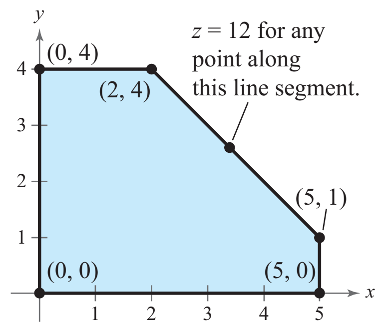


Figure E.17

In this case, you can conclude that the objective function has a maximum value (of 12) not only at the vertices (2, 4) and (5, 1), but also at *any point on the line segment connecting these two vertices*, as shown in Figure E.17. Note that by rewriting the objective function as

$$y = -x + \frac{1}{2}z$$

you can see that its graph has the same slope as the line through the vertices (2, 4) and (5, 1).

Some linear programming problems have no optimal solutions. This can occur if the region determined by the constraints is *unbounded*.

Example 4 An Unbounded Region

Find the maximum value of

$z = 4x + 2y$ Objective function

where $x \geq 0$ and $y \geq 0$, subject to the following constraints.

$$\left.\begin{aligned} x + 2y &\geq 4 \\ 3x + y &\geq 7 \\ -x + 2y &\leq 7 \end{aligned}\right\}$$ Constraints

Solution

The region determined by the constraints is shown in Figure E.18. For this unbounded region, there is no maximum value of z. To see this, note that the point $(x, 0)$ lies in the region for all values of $x \geq 4$. By choosing large values of x, you can obtain values of $z = 4(x) + 2(0) = 4x$ that are as large as you want. So, there is no maximum value of z. For the vertices of the region, the objective function has the following values. So, there *is* a minimum value of z, $z = 10$, which occurs at the vertex (2, 1).

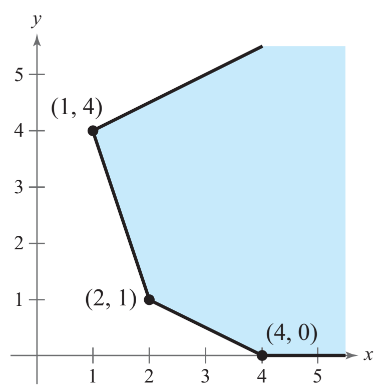


Figure E.18

At (1, 4): $z = 4(1) + 2(4) = 12$

At (2, 1): $z = 4(2) + 2(1) = 10$ Minimum value of z

At (4, 0): $z = 4(4) + 2(0) = 16$

CHECKPOINT Now try Exercise 31.

Applications

Example 5 shows how linear programming can be used to find the maximum profit in a business application.

Example 5 Optimizing Profit

A manufacturer wants to maximize the profit from selling two types of boxed chocolates. A box of chocolate covered creams yields a profit of \$1.50, and a box of chocolate covered cherries yields a profit of \$2.00. Market tests and available resources have indicated the following constraints.

1. The combined production level should not exceed 1200 boxes per month.
2. The demand for a box of chocolate covered cherries is no more than half the demand for a box of chocolate covered creams.
3. The production level of a box of chocolate covered creams is less than or equal to 600 boxes plus three times the production level of a box of chocolate covered cherries.

Solution

Let x be the number of boxes of chocolate covered creams and y be the number of boxes of chocolate covered cherries. The objective function (for the combined profit) is given by

$P = 1.5x + 2y.$ Objective function

The three constraints translate into the following linear inequalities.

1. $x + y \leq 1200$ ⇒ $x + y \leq 1200$

2. $y \leq \frac{1}{2}x$ ⇒ $-x + 2y \leq 0$

3. $x \leq 3y + 600$ ⇒ $x - 3y \leq 600$

Because neither x nor y can be negative, you also have the two additional constraints of $x \geq 0$ and $y \geq 0$. Figure E.19 shows the region determined by the constraints. To find the maximum profit, test the value of P at each vertex of the region.

At (0, 0): $P = 1.5(0) + 2(0) = 0$

At (800, 400): $P = 1.5(800) + 2(400) = 2000$ Maximum profit

At (1050, 150): $P = 1.5(1050) + 2(150) = 1875$

At (600, 0): $P = 1.5(600) + 2(0) = 900$

So, the maximum profit is \$2000, and it occurs when the monthly production consists of 800 boxes of chocolate covered creams and 400 boxes of chocolate covered cherries.

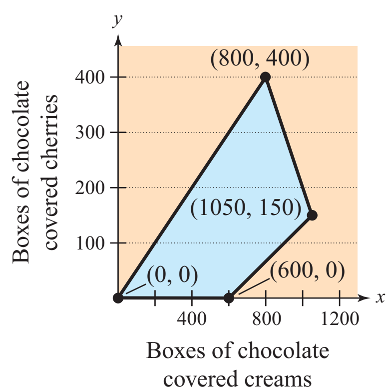


Figure E.19

CHECKPOINT Now try Exercise 35.

In Example 5, suppose the manufacturer improves the production of chocolate covered creams so that a profit of \$2.50 per box is obtained. The maximum profit can now be found using the objective function $P = 2.5x + 2y$. By testing the values of P at the vertices of the region, you find that the maximum profit is now \$2925, which occurs when $x = 1050$ and $y = 150$.

Example 6 Optimizing Cost

The minimum daily requirements from the liquid portion of a diet are 300 calories, 36 units of vitamin A, and 90 units of vitamin C. A cup of dietary drink X costs \$0.12 and provides 60 calories, 12 units of vitamin A, and 10 units of vitamin C. A cup of dietary drink Y costs \$0.15 and provides 60 calories, 6 units of vitamin A, and 30 units of vitamin C. How many cups of each drink should be consumed each day to minimize the cost and still meet the daily requirements?

Solution

As in Example 9 on page A52, let x be the number of cups of dietary drink X and let y be the number of cups of dietary drink Y.

$$\left.\begin{array}{lr} \textit{For Calories:} & 60x + 60y \geq 300 \\ \textit{For Vitamin A:} & 12x + 6y \geq 36 \\ \textit{For Vitamin C:} & 10x + 30y \geq 90 \\ & x \geq 0 \\ & y \geq 0 \end{array}\right\} \quad \text{Constraints}$$

The cost C is given by

$$C = 0.12x + 0.15y. \quad \text{Objective function}$$

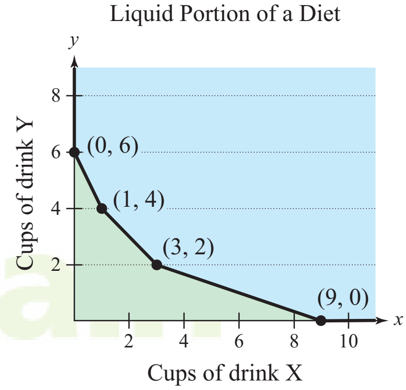


Figure E.20

The graph of the region determined by the constraints is shown in Figure E.20. To determine the minimum cost, test C at each vertex of the region.

At (0, 6): $C = 0.12(0) + 0.15(6) = 0.90$

At (1, 4): $C = 0.12(1) + 0.15(4) = 0.72$

At (3, 2): $C = 0.12(3) + 0.15(2) = 0.66$ Minimum value of C

At (9, 0): $C = 0.12(9) + 0.15(0) = 1.08$

So, the minimum cost is \$0.66 per day, and this cost occurs when three cups of drink X and two cups of drink Y are consumed each day.

CHECKPOINT Now try Exercise 37.

TECHNOLOGY TIP You can check the points of the vertices of the constraints by using a graphing utility to graph the equations that represent the boundaries of the inequalities. Then use the *intersect* feature to confirm the vertices.

E.2 Exercises

See www.CalcChat.com for worked-out solutions to odd-numbered exercises.

Vocabulary Check

Fill in the blanks.

1. In the process called _______ , you are asked to find the minimum or maximum value of a quantity.

2. The _______ of a linear programming problem gives the quantity that is to be maximized or minimized.

3. The ________ of a linear programming problem determine the set of _______ .

In Exercises 1–12, find the minimum and maximum values of the objective function and where they occur, subject to the indicated constraints. (For each exercise, the graph of the region determined by the constraints is provided.)

1. Objective function:

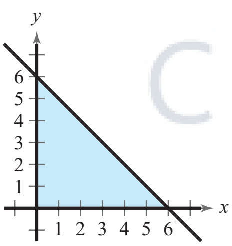
$z = 3x + 5y$

Constraints:

$x \geq 0$

$y \geq 0$

$x + y \leq 6$



2. Objective function:

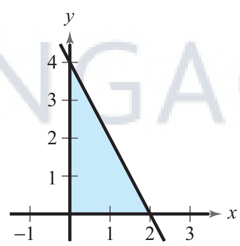
$z = 2x + 8y$

Constraints:

$x \geq 0$

$y \geq 0$

$2x + y \leq 4$



3. Objective function:

$z = 10x + 7y$

Constraints:
See Exercise 1.

4. Objective function:

$z = 7x + 3y$

Constraints:
See Exercise 2.

5. Objective function:

$z = 3x + 2y$

Constraints:

$x \geq 0$

$y \geq 0$

$x + 3y \leq 15$

$4x + y \leq 16$

6. Objective function:

$z = 4x + 3y$

Constraints:

$x \geq 0$

$2x + 3y \geq 6$

$3x - 2y \leq 9$

$x + 5y \leq 20$

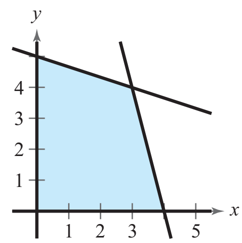


Figure for 5

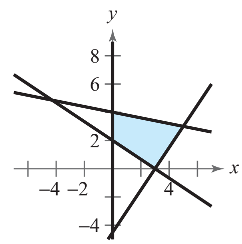


Figure for 6

7. Objective function:

$z = 5x + 0.5y$

Constraints:
See Exercise 5.

8. Objective function:

$z = x + 6y$

Constraints:
See Exercise 6.

9. Objective function:

$z = 10x + 7y$

Constraints:

$0 \leq x \leq 60$

$0 \leq y \leq 45$

$5x + 6y \leq 420$

10. Objective function:

$z = 50x + 35y$

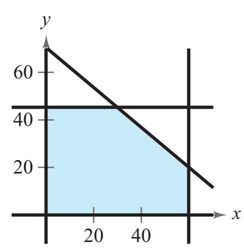
Constraints:

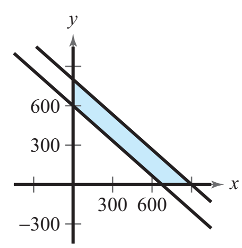
$x \geq 0$

$y \geq 0$

$8x + 9y \leq 7200$

$8x + 9y \geq 5400$





11. Objective function:

$z = 25x + 30y$

Constraints:
See Exercise 9.

12. Objective function:

$z = 15x + 20y$

Constraints:
See Exercise 10.

In Exercises 13–26, sketch the region determined by the constraints. Then find the minimum and maximum values of the objective function and where they occur, subject to the indicated constraints.

13. Objective function:

$z = 6x + 10y$

Constraints:

$x \geq 0$

$y \geq 0$

$2x + 5y \leq 10$

14. Objective function:

$z = 7x + 8y$

Constraints:

$x \geq 0$

$y \geq 0$

$x + \frac{1}{2}y \leq 4$

15. Objective function:

$z = 3x + 4y$

Constraints:

$x \geq 0$

$y \geq 0$

$2x + 5y \leq 50$

$4x + y \leq 28$

16. Objective function:

$z = 4x + 5y$

Constraints:

$x \geq 0$

$y \geq 0$

$2x + 2y \leq 10$

$x + 2y \leq 6$

17. Objective function:

$z = x + 2y$

Constraints:
See Exercise 15.

18. Objective function:

$z = 2x + 4y$

Constraints:
See Exercise 16.

19. Objective function:

$z = 2x$

Constraints:
See Exercise 15.

20. Objective function:

$z = 3y$

Constraints:
See Exercise 16.

21. Objective function:

$z = 4x + y$

Constraints:

$x \geq 0$

$y \geq 0$

$x + 2y \leq 40$

$2x + 3y \geq 72$

22. Objective function:

$z = x$

Constraints:

$x \geq 0$

$y \geq 0$

$2x + 3y \leq 60$

$2x + y \leq 28$

$4x + y \leq 48$

23. Objective function:

$z = x + 4y$

Constraints:
See Exercise 21.

24. Objective function:

$z = y$

Constraints:
See Exercise 22.

25. Objective function:

$z = 2x + 3y$

Constraints:
See Exercise 21.

26. Objective function:

$z = 3x + 2y$

Constraints:
See Exercise 22.

Exploration **In Exercises 27–30, perform the following.**

(a) Graph the region bounded by the following constraints.

$3x + y \leq 15$

$4x + 3y \leq 30$

$x \geq 0$

$y \geq 0$

(b) Graph the objective function for the given maximum value of z on the same set of coordinate axes as the graph of the constraints.

(c) Use the graph to determine the feasible point or points that yield the maximum. Explain how you arrived at your answer.

	Objective Function	*Maximum*
27.	$z = 2x + y$	$z = 12$
28.	$z = 5x + y$	$z = 25$
29.	$z = x + y$	$z = 10$
30.	$z = 3x + y$	$z = 15$

In Exercises 31–34, the linear programming problem has an unusual characteristic. Sketch a graph of the solution region for the problem and describe the unusual characteristic. The objective function is to be maximized in each case.

31. Objective function:

$z = x + y$

Constraints:

$x \geq 0$

$y \geq 0$

$-x + y \leq 1$

$-x + 2y \leq 4$

32. Objective function:

$z = 2.5x + y$

Constraints:

$x \geq 0$

$y \geq 0$

$3x + 5y \leq 15$

$5x + 2y \leq 10$

33. Objective function:

$z = x + y$

Constraints:

$x \geq 0$

$y \geq 0$

$-x + y \leq 0$

$-3x + y \geq 3$

34. Objective function:

$z = -x + 2y$

Constraints:

$x \geq 0$

$y \geq 0$

$x \leq 10$

$x + y \leq 7$

35. ***Optimizing Revenue*** An accounting firm has 800 hours of staff time and 96 hours of reviewing time available each week. The firm charges \$2000 for an audit and \$300 for a tax return. Each audit requires 100 hours of staff time and 8 hours of review time. Each tax return requires 12.5 hours of staff time and 2 hours of review time. (a) What numbers of audits and tax returns will yield the maximum revenue? (b) What is the maximum revenue?

36. ***Optimizing Profit*** A manufacturer produces two models of snowboards. The amounts of time (in hours) required for assembling, painting, and packaging the two models are as follows.

	Model A	Model B
Assembling	2.5	3
Painting	2	1
Packaging	0.75	1.25

The total amounts of time available for assembling, painting, and packaging are 4000 hours, 2500 hours, and 1500 hours, respectively. The profits per unit are \$50 for model A and \$52 for model B.

(a) How many of each model should be produced to maximize profit?

(b) What is the maximum profit?

37. ***Optimizing Cost*** A farming cooperative mixes two brands of cattle feed. Brand X costs \$25 per bag and contains two units of nutritional element A, two units of nutritional element B, and two units of nutritional element C. Brand Y costs \$20 per bag and contains one unit of nutritional element A, nine units of nutritional element B, and three units of nutritional element C. The minimum requirements for nutritional elements A, B, and C are 12 units, 36 units, and 24 units, respectively.

(a) Find the number of bags of each brand that should be mixed to produce a mixture having a minimum cost per bag.

(b) What is the minimum cost?

38. ***Optimizing Cost*** A pet supply company mixes two brands of dry dog food. Brand X costs \$15 per bag and contains eight units of nutritional element A, one unit of nutritional element B, and two units of nutritional element C. Brand Y costs \$30 per bag and contains two units of nutritional element A, one unit of nutritional element B, and seven units of nutritional element C. Each bag of mixed dog food must contain at least 16 units, 5 units, and 20 units of nutritional elements A, B, and C, respectively.

(a) Find the numbers of bags of brands X and Y that should be mixed to produce a mixture meeting the minimum nutritional requirements and having a minimum cost per bag.

(b) What is the minimum cost?

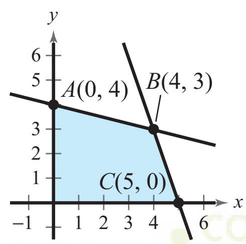
Synthesis

True or False? **In Exercises 39 and 40, determine whether the statement is true or false. Justify your answer.**

39. If an objective function has a maximum value at the adjacent vertices (4, 7) and (8, 3), you can conclude that it also has a maximum value at the points (4.5, 6.5) and (7.8, 3.2).

40. When solving a linear programming problem, if the objective function has a maximum value at two adjacent vertices, you can assume that there are an infinite number of points that will produce the maximum value.

Think About It **In Exercises 41–44, find an objective function that has a maximum or minimum value at the indicated vertex of the constraint region shown below. (There are many correct answers.)**



41. The maximum occurs at vertex A.

42. The maximum occurs at vertex B.

43. The maximum occurs at vertex C.

44. The minimum occurs at vertex C.

In Exercises 45 and 46, determine values of t such that the objective function has a maximum value at each indicated vertex.

45. Objective function:

$z = 3x + ty$

Constraints:

$$x \geq 0$$
$$y \geq 0$$
$$x + 3y \leq 15$$
$$4x + y \leq 16$$

(a) (0, 5)

(b) (3, 4)

46. Objective function:

$z = 3x + ty$

Constraints:

$$x \geq 0$$
$$y \geq 0$$
$$x + 2y \leq 4$$
$$x - y \leq 1$$

(a) (2, 1)

(b) (0, 2)

Appendix F: Study Capsules

Study Capsule 1 Algebraic Expressions and Functions

Properties

Exponents and Radicals

Properties of Exponents

1. $a^m \cdot a^n = a^{m+n}$ **2.** $\dfrac{a^m}{a^n} = a^{m-n}$ **3.** $(a^m)^n = a^{mn}$ **4.** $a^{-n} = \dfrac{1}{a^n}; \ \dfrac{1}{a^{-n}} = a^n$ **5.** $a^0 = 1, \ a \neq 0$

Properties of Radicals

1. $\sqrt{a \cdot b} = \sqrt{a} \cdot \sqrt{b}$ **2.** $\sqrt{\dfrac{a}{b}} = \dfrac{\sqrt{a}}{\sqrt{b}}$ **3.** $\sqrt{a^2} = |a|$ **4.** $\sqrt[n]{a} = a^{1/n}$ **5.** $\sqrt[n]{a^m} = a^{m/n} = \left(\sqrt[n]{a}\right)^m, \ a > 0$

Methods

Polynomials and Factoring

Factoring Quadratics

1. $x^2 + bx + c = (x + \square)(x + \square)$

Fill blanks with factors of c that add up to b.

2. $ax^2 + bx + c = (\square x + \square)(\square x + \square)$

Fill blanks with factors of a and of c, so that the binomial product has a middle factor of bx.

Factoring Polynomials

Factor a polynomial $ax^3 + bx^2 + cx + d$ by grouping.

Examples

$x^2 - 7x + 12 = (x + \square)(x + \square)$ — Factor 12 as $(-3)(-4)$.

$= (x - 3)(x - 4)$

$4x^2 + 4x - 15 = (\square x + \square)(\square x + \square)$ — Factors of 4 (first blanks); Factors of -15 (second blanks)

$= (2x - 3)(2x + 5)$ — Factor 4 as (2)(2). Factor -15 as $(-3)(5)$.

$4x^3 + 12x^2 - x - 3$

$= (4x^3 + 12x^2) - (x + 3)$ — Group by pairs.

$= 4x^2(x + 3) - (x + 3)$ — Factor out monomial.

$= (x + 3)(4x^2 - 1)$ — Factor out binomial.

$= (x + 3)(2x + 1)(2x - 1)$ — Difference of squares

Simplifying Expressions

Fractional Expressions

1. Factor completely and simplify.

$\dfrac{2x^3 - 4x^2 - 6x}{2x^2 - 18} = \dfrac{2x(x^2 - 2x - 3)}{2(x^2 - 9)}$ — Factor out monomials.

$= \dfrac{2x(x - 3)(x + 1)}{2(x + 3)(x - 3)}$ — Factor quadratics.

$= \dfrac{x(x + 1)}{x + 3}, \ x \neq 3$ — Divide out common factors.

2. Rationalize denominator. (*Note:* Radicals in the numerator can be rationalized in a similar manner.)

$\dfrac{3x}{\sqrt{x - 5} + 2} = \dfrac{3x}{\sqrt{x - 5} + 2} \cdot \dfrac{\sqrt{x - 5} - 2}{\sqrt{x - 5} - 2}$ — Multiply by conjugate.

$= \dfrac{3x\left(\sqrt{x - 5} - 2\right)}{(x - 5) - 4}$ — Difference of squares

$= \dfrac{3x\left(\sqrt{x - 5} - 2\right)}{x - 9}$ — Simplify.

Study Capsule 1 Algebraic Expressions and Functions (continued)

Equations and Graphs

Equations

Slope of a Line Passing Through (x_1, y_1) and (x_2, y_2):

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$m_1 = m_2$	Parallel lines
$m_1 = -\dfrac{1}{m_2}$	Perpendicular lines

Equations of Lines

$y = mx + b$	Slope-intercept form
$y - y_1 = m(x - x_1)$	Point-slope form
$Ax + By + C = 0$	General form
$x = a,\ y = b$	Vertical and horizontal lines

Distance Between Points (x_1, y_1) and (x_2, y_2):

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Graphs

Graphing Equations by Point Plotting

1. Write equation in the form $y =$. . . .
2. Make a table of values.
3. Find intercepts.
4. Use symmetry.
5. Plot points and connect with smooth curve.

Graphing Equations with a Graphing Utility

1. Enter the equation in the form $y =$. . . .
2. Identify domain and range.
3. Set an appropriate viewing window.

Functions and Graphs

Functions

Definition: f is a function if to each element x in the domain of f there corresponds exactly one element y in the range of f.

Notation: $y = f(x)$

f is the name of the function.

y is the dependent variable, or the output value.

x is the independent variable, or the input value.

$f(x)$ is the value of the function at x.

Examples

Polynomial Function: $f(x) = 2x^3 - 3x^2 - 4x + 6$

Piecewise-Defined Function: $f(x) = \begin{cases} 2 - 3x, & x > 1 \\ x^2 + 2x, & x \le 1 \end{cases}$

Inverse Functions f and f^{-1}: Their graphs are reflections of each other in the line $y = x$.

$$f(f^{-1}(x)) = x \quad \text{and} \quad f^{-1}(f(x)) = x$$

To find the inverse function of $y = f(x)$, if it exists, interchange x and y, then solve for y. The result is $f^{-1}(x)$.

Transformations and Compositions

Transformations

Transformations of the Graph of $y = f(x)$

Vertical shifts:	$h(x) = f(x) + c$	Upward c units
	$h(x) = f(x) - c$	Downward c units
Horizontal shifts:	$h(x) = f(x - c)$	Right shift c units
	$h(x) = f(x + c)$	Left shift c units
Reflections:	$h(x) = -f(x)$	Reflection in x-axis
	$h(x) = f(-x)$	Reflection in y-axis
Stretches/Shrinks:	$h(x) = cf(x)$	Vertical stretch, $c > 1$
		Vertical shrink, $c < 1$
	$h(x) = f(cx)$	Horizontal stretch, $0 < c < 1$
		Horizontal shrink, $c > 1$

Compositions

Compositions of Functions

$(f \circ g)(x) = f(g(x))$

$(g \circ f)(x) = g(f(x))$

Examples

$f(x) = x^2, \quad g(x) = 2x - 1$

$$\begin{aligned}(f \circ g)(x) &= f(g(x)) \\ &= f(2x - 1) \\ &= (2x - 1)^2 \\ &= 4x^2 - 4x + 1\end{aligned}$$

$$\begin{aligned}(g \circ f)(x) &= g(f(x)) \\ &= g(x^2) \\ &= 2(x^2) - 1 \\ &= 2x^2 - 1\end{aligned}$$

Study Capsule 2 Graphing Algebraic Functions

Linear Functions

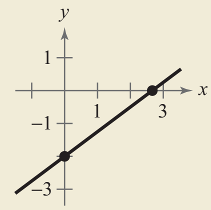
Graphical Analysis

Graph of $f(x) = mx + b$ is a line.

1. m = slope of a line
2. y-intercept: $(0, b)$
3. x-intercept: $(k, 0)$, where k is solution to $0 = mx + b$

Example

To graph the linear equation $3x - 4y - 8 = 0$, solve for y to get $y = \frac{3}{4}x - 2$.
So, $m = \frac{3}{4}$, the y-intercept is $(0, -2)$, and the x-intercept is $\left(\frac{8}{3}, 0\right)$.



Quadratic Functions

Graphical Analysis

Graph of $y = ax^2 + bx + c$ is a parabola (U-shaped).

1. Opens upward if $a > 0$.
   Opens downward if $a < 0$.
2. Vertex: $\left(-\frac{b}{2a}, f\left(-\frac{b}{2a}\right)\right)$
3. Vertex is minimum if $a > 0$.
   Vertex is maximum if $a < 0$.
4. Axis of symmetry: $x = -\frac{b}{2a}$

Example

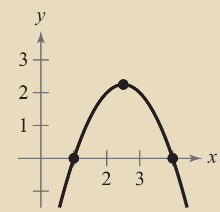
$y = -x^2 + 5x - 4$ opens downward because $a = -1$.

Vertex:

$$\left(\frac{-5}{2(-1)}, f\left(\frac{-5}{2(-1)}\right)\right) = \left(\frac{5}{2}, \frac{9}{4}\right)$$

Vertex is a maximum.

Axis of symmetry is $x = \frac{5}{2}$.



Polynomial Functions

Graphical Analysis

Graph of $f(x) = a_nx^n + a_{n-1}x^{n-1} + \cdot\cdot\cdot + a_1x + a_0$ has the following characteristics.

1. x-intercepts occur at zeros of f.
   y-intercept is $(0, a_0)$.
2. Right-hand and left-hand behaviors:

	$a_n > 0$	$a_n < 0$
n is odd	Falls to left, rises to right	Rises to left, falls to right
n is even	Rises to left and right	Falls to left and right

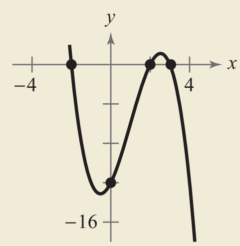
Example

$y = -x^3 + 3x^2 + 4x - 12$
$= (x - 3)(4 - x^2)$

x-intercepts: $(\pm 2, 0)$, $(3, 0)$

y-intercept: $(0, -12)$

End behavior: Up to left and down to right because $a_n < 0$ and n is odd.



Rational Functions

Graphical Analysis

Graph of

$$f(x) = \frac{N(x)}{D(x)} = \frac{a_nx^n + a_{n-1}x^{n-1} + \cdot\cdot\cdot + a_1x + a_0}{b_mx^m + b_{m-1}x^{m-1} + \cdot\cdot\cdot + b_1x + b_0}$$

where N and D have no common factors, has the following characteristics.

1. x-intercepts occur at zeros of $N(x)$.
2. Vertical asymptotes occur at zeros of $D(x)$.
3. Horizontal asymptote occurs at $y = 0$ when $n < m$, and at $y = a_n/b_m$ when $n = m$.

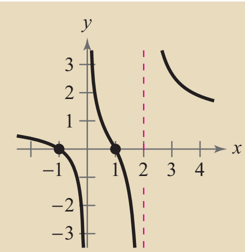
Example

$$y = \frac{x^2 - 1}{x^2 - 2x} = \frac{(x + 1)(x - 1)}{x(x - 2)}$$

x-intercepts: $(\pm 1, 0)$

Vertical asymptotes: $x = 0$, $x = 2$

Horizontal asymptote: $y = \frac{1}{1} = 1$



Study Capsule 3 Zeros of Algebraic Functions

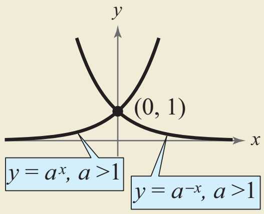
	Solution Strategy	Examples
Linear Functions	Solve $ax + b = c$ by isolating x using *inverse* operations.	$-3x + 5 = 8$ Original equation $-3x = 3$ First, subtract 5 from each side. $x = \frac{3}{-3} = -1$ Then, divide each side by -3.
Quadratic Functions	Solve for $ax^2 + bx + c = 0$ using one of the following methods. **1.** Factor. **2.** Use the Quadratic Formula $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$. **3.** Complete the square and/or extract square roots.	$2x^2 + 5x - 3 = 0$ **1.** $(2x - 1)(x + 3) = 0$ Factor. $2x - 1 = 0$ $x + 3 = 0$ Set factors equal to 0. $x = \frac{1}{2}$ or $x = -3$ Solve for x. **2.** $x = \dfrac{-5 \pm \sqrt{5^2 - 4(2)(-3)}}{2(2)}$ $a = 2, b = 5, c = -3$ $x = \dfrac{-5 \pm \sqrt{49}}{4} = \dfrac{-5 \pm 7}{4} \Rightarrow x = \frac{1}{2}, -3$
Polynomial Functions	Solve $a_nx^n + a_{n-1}x^{n-1} + \cdot \cdot \cdot + a_1x + a_0 = 0$ by using the Rational Zero Test in combination with synthetic division. Possible rational zeros $= \dfrac{\pm \text{ factors of } a_0}{\pm \text{ factors of } a_n}$ *Note:* To solve a polynomial inequality, find the zeros of the corresponding equation and test the inequality between and beyond each zero.	$x^3 + x^2 - 4x - 4 = 0$, where $a_n = 1$ and $a_0 = -4$. Possible rational zeros are $\pm1, \pm2$, and ±4, so try $x = -1$: $f(-1) = (-1)^3 + (-1)^2 - 4(-1) - 4 = 0$ Synthetic division using the zero $x = -1$ $\begin{array}{r\|rrrr} -1 & 1 & 1 & -4 & -4 \\ & & -1 & 0 & 4 \\ \hline & 1 & 0 & -4 & 0 \end{array} \Rightarrow x^2 - 4 = 0$ So, the zeros are -1 and ±2.
Other Functions	**1.** Solve an equation involving *radicals* (or fractional powers) by isolating the radical and then raising each side to the appropriate power to obtain a polynomial equation. **2.** Solve an equation involving *fractions* by multiplying each side by the LCD of the fractions to obtain a polynomial equation. **3.** Solve an *absolute value* equation, $\|f(x)\| = g(x)$, by solving for x in the two equations $f(x) = g(x)$ and $-f(x) = g(x)$. **4.** To solve $\|f(x)\| \le c$, isolate x in $-c \le f(x) \le c$. **5.** To solve $\|f(x)\| \ge c$, isolate x in both $f(x) \ge c$ and $f(x) \le -c$.	**1.** $2\sqrt{x + 3} - x = 4$ Original equation $2\sqrt{x + 3} = x + 4$ Isolate radical term. $4(x + 3) = (x + 4)^2$ Raise each side to 2nd power. $0 = x^2 + 4x + 4$ Standard form $0 = (x + 2)^2$ $x = -2$ is repeated zero. **2.** $6 + \dfrac{2}{x + 3} = \dfrac{6x + 1}{3}$ Original equation $6(3)(x + 3) + 2(3) = (6x + 1)(x + 3)$ Multiply by LCD. $0 = 6x^2 + x - 57$ Standard form $0 = (6x + 19)(x - 3)$ Factor. **3.** $x^2 - 5x = \|x - 5\|$ Isolate absolute value. $x^2 - 5x = (x - 5)$ or $x^2 - 5x = -(x - 5)$ $x^2 - 6x + 5 = 0$ $x^2 - 4x - 5 = 0$ $(x - 5)(x - 1) = 0$ $(x - 5)(x + 1) = 0$ $x = 5, x = 1$ $x = 5, x = -1$ *Note:* The only solutions are $x = 5$ and $x = -1$.

Study Capsule 4 Exponential and Logarithmic Functions

Definitions and Graphs

Exponential Functions

Definition: The exponential function f with base a is denoted by $f(x) = a^x$, where $a > 0$, $a \neq 1$, and x is any real number.



Domain: $(-\infty, \infty)$

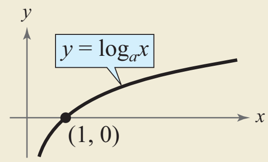
Range: $(0, \infty)$

Intercept: $(0, 1)$

Horizontal asymptote: $y = 0$

Logarithmic Functions

Definition: For $x > 0$, $a > 0$, and $a \neq 1$, the logarithmic function f with base a is $f(x) = \log_a x$, where $y = \log_a x$ if and only if $x = a^y$.



Domain: $(0, \infty)$

Range: $(-\infty, \infty)$

Intercept: $(1, 0)$

Vertical asymptote: $x = 0$

Properties

Exponential Functions

One-to-One: $a^x = a^y \implies x = y$

Inverse: $a^{\log_a x} = x$

Product: $a^u \cdot a^v = a^{u+v}$

Quotient: $\dfrac{a^u}{a^v} = a^{u-v}$

Power: $(a^u)^v = a^{u \cdot v}$

Others: $a^0 = 1$

Note: The same properties hold for the natural base e, where e is the constant 2.718281828 . . . .

Logarithmic Functions

One-to-One: $\log_a x = \log_a y \implies x = y$

Inverse: $\log_a a^x = x$ ***Power:*** $\log_a u^v = v \log_a u$

Product: $\log_a(u \cdot v) = \log_a u + \log_a v$

Quotient: $\log_a\left(\dfrac{u}{v}\right) = \log_a u - \log_a v$

Others: $\log_a a = 1$; $\log_a 1 = 0$; $\log_a 0$ is undefined.

Change of Base: $\log_a x = \dfrac{\log_b x}{\log_b a} = \dfrac{\log_{10} x}{\log_{10} a} = \dfrac{\ln x}{\ln a}$

Note: The same properties apply for bases 10 and e.

Solving Equations

Exponential Functions

Solve an exponential equation by isolating the exponential term and taking the logarithm of each side.

$2^x - 5 = 0$	Original equation
$2^x = 5$	Isolate exponential term.
$\log_2 2^x = \log_2 5$	Take log of each side.
$x = \log_2 5$	Inverse Property
$x = \dfrac{\ln 5}{\ln 2}$	Change-of-base formula
$x \approx 2.32$	Use a calculator.

Some exponential equations can be solved by using the Inverse Property.

$3e^{2x} - 2 = 5$	Original equation
$3e^{2x} = 7$	Add 2 to each side.
$e^{2x} = \frac{7}{3}$	Isolate exponential term.
$\ln e^{2x} = \ln \frac{7}{3}$	Take natural log of each side.
$2x = \ln \frac{7}{3}$	Inverse Property
$x = \frac{1}{2} \ln \frac{7}{3}$	Multiply each side by $\frac{1}{2}$.
$x \approx 0.42$	Use a calculator.

Logarithmic Functions

Solve a logarithmic equation by isolating the logarithmic term and exponentiating each side.

$6 + 2\log_{10} x = 3$	Original equation
$\log_{10} x = -\frac{3}{2}$	Isolate logarithmic term.
$10^{\log_{10} x} = 10^{-3/2}$	Exponentiate using base 10.
$x = 10^{-3/2}$	Inverse Property
$x \approx 0.0316$	Use a calculator.

Properties of logarithms are useful in rewriting equations in forms that are easier to solve.

$\ln(x + 4) - \ln(x - 2) = \ln x$	Original equation
$\ln\left(\dfrac{x + 4}{x - 2}\right) = \ln x$	Quotient Property
$\dfrac{x + 4}{x - 2} = x$	One-to-One Property
$x + 4 = x^2 - 2x$	
$0 = x^2 - 3x - 4$	Standard form
$0 = (x - 4)(x + 1)$	Factor.

$x = 4$ is a valid solution. $x = -1$ is not in the domain of $\ln(x - 2)$.

Study Capsule 5 Linear Systems and Matrices

Substitution

Systems of Equations

Needed for problems that involve two or more equations in two or more variables.

Method of Substitution: Solve for one variable in terms of the other. Substitute this expression into the other equation and solve this one-variable equation. Back-substitute to find the value of the other variable.

Examples

Linear:

$$\begin{cases} 2x + y = 1 \\ x - y = 5 \end{cases} \implies y = x - 5$$

$2x + (x - 5) = 1 \implies x = 2$

$2 - y = 5 \implies y = -3$

Nonlinear:

$$\begin{cases} x - y^2 = 1 \\ x - y = 3 \end{cases} \implies y = x - 3$$

$x - (x - 3)^2 = 1$

$x^2 - 7x + 10 = 0$

$(x - 5)(x - 2) = 0$

$x = 5, y = 2$ and $x = 2, y = -1$

Graphical Interpretation

Linear:

Lines intersect $\implies$ one solution

Lines parallel $\implies$ no solution

Lines coincide $\implies$ infinite number of solutions

Nonlinear:

Graphs intersect at one point.

Graphs intersect at multiple points.

Graphs do not intersect.

Elimination

Algebraic Methods

Method of Elimination: Obtain coefficients for x (or y) that differ only in sign by multiplying one or both equations by appropriate constants. Then add the equations to eliminate one variable. Solve the remaining one-variable equation. Back-substitute into one of the original equations to find the value of the other variable.

$$\begin{cases} 2x + y = 3 \\ 4x + 3y = -1 \end{cases} \Rightarrow \begin{array}{rcl} -6x - 3y &=& -9 \\ 4x + 3y &=& -1 \\ \hline -2x &=& -10 \\ x &=& 5, y = -7 \end{array}$$

Gaussian Elimination: For systems of linear equations in more than two variables, use elementary row operations to rewrite the system in row-echelon form. Back-substitute into one of the original equations to find the value of each remaining variable.

$$\begin{cases} x + 2y + z = 3 \\ 2x + 5y - z = -4 \\ 3x - 2y - z = 5 \end{cases} \begin{array}{l} \rightarrow \\ -2R_1 + R_2 \rightarrow \\ -3R_1 + R_3 \rightarrow \end{array} \begin{cases} x + 2y + z = 3 \\ y - 3z = -10 \\ -8y - 4z = -4 \end{cases}$$

Using $8R_2 + R_3$ for row 3, the row-echelon form is

$$\begin{cases} x + 2y + z = 3 \\ y - 3z = -10 \\ z = 3 \end{cases}.$$

Back-substitution yields $y = -1$ and $x = 2$.

Types of systems:

Consistent and independent, if one solution

Consistent and dependent, if infinitely many solutions

Inconsistent, if no solution

Matrix Methods

Gauss-Jordan Elimination: Form the augmented matrix for a system of equations and apply elementary row operations until a reduced row-echelon matrix is obtained.

Augmented Matrix *Reduced Row-Echelon*

$$\begin{bmatrix} 1 & 1 & 1 & \vdots & -1 \\ 3 & 5 & 4 & \vdots & 2 \\ 3 & 6 & 5 & \vdots & 0 \end{bmatrix} \quad \begin{bmatrix} 1 & 0 & 0 & \vdots & 1 \\ 0 & 1 & 0 & \vdots & 7 \\ 0 & 0 & 1 & \vdots & -9 \end{bmatrix}$$

Solution: $(1, 7, -9)$

Solve a Matrix Equation: Solve the matrix equation $AX = B$, using the inverse A^{-1} to obtain $X = A^{-1}B$. The inverse of A is found by converting the matrix $[A \vdots I]$ into the form $[I \vdots A^{-1}]$, where I is the identity matrix.

$$AX = B \qquad \begin{bmatrix} 1 & 1 & 1 \\ 3 & 5 & 4 \\ 3 & 6 & 5 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -1 \\ 2 \\ 0 \end{bmatrix}$$

$$X = A^{-1}B \qquad \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 & 1 & -1 \\ -3 & 2 & -1 \\ 3 & -3 & 2 \end{bmatrix} \begin{bmatrix} -1 \\ 2 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 7 \\ -9 \end{bmatrix}$$

Cramer's Rule: $x = \dfrac{|A_1|}{|A|}$, $y = \dfrac{|A_2|}{|A|}$, $z = \dfrac{|A_3|}{|A|}$, where

$$|A| = \begin{vmatrix} 1 & 1 & 1 \\ 3 & 5 & 4 \\ 3 & 6 & 5 \end{vmatrix}, \quad |A_1| = \begin{vmatrix} -1 & 1 & 1 \\ 2 & 5 & 4 \\ 0 & 6 & 5 \end{vmatrix},$$

$$|A_2| = \begin{vmatrix} 1 & -1 & 1 \\ 3 & 2 & 4 \\ 3 & 0 & 5 \end{vmatrix}, \quad |A_3| = \begin{vmatrix} 1 & 1 & -1 \\ 3 & 5 & 2 \\ 3 & 6 & 0 \end{vmatrix}$$

Study Capsule 6 Sequences, Series, and Probability

Sequences

General

Definition: An infinite sequence $\{a_n\}$ has function values $a_1, a_2, a_3, \ldots, a_n, \ldots$ called the *terms* of the sequence.

Skills: Use or find the form of the *n*th term

1. Given the form of a_n, write the first five terms.
2. Given the first several terms, find a_n.

Arithmetic

Definition: $\{a_n\}$ is *arithmetic* if the *difference* between consecutive terms is a common value d.

Skills: Given d and a_1, or given two specific terms

1. Find the first five terms of $\{a_n\}$.
2. Find the form of a_n.

In general, $a_n = a_1 + (n - 1)d$.

Geometric

Definition: $\{a_n\}$ is *geometric* if the *ratio* of two consecutive terms is a common value r.

Skills: Given r and a_1, or given two specific terms

1. Find the first five terms of $\{a_n\}$.
2. Find the form of a_n.

In general, $a_n = a_1 r^{n-1}$.

Sums and Series

General

Summation Notation: $\sum_{i=1}^{n} a_i$

There is no general formula for calculating the *n*th partial sum or the sum of an infinite series.

Arithmetic

nth Partial Sum:

$$S_n = \sum_{i=1}^{n} a_i = \frac{n}{2}(a_1 + a_n)$$

where $a_n = a_1 + (n - 1)d$

Infinite Series:

$$S = \sum_{n=1}^{\infty} a_n = \text{[sum is not finite]}$$

Geometric

nth Partial Sum:

$$S_n = \sum_{i=1}^{n} a_i = a_1\left(\frac{1 - r^n}{1 - r}\right), \quad r \neq 1$$

Infinite Series:

$$S = \sum_{n=1}^{\infty} a_n = \frac{a_1}{1 - r}, \quad |r| < 1$$

Binomial Theorem, Counting Principles, and Probability

Binomial Theorem

Binomial Theorem:

$(x + y)^n = x^n + nx^{n-1}y + \cdots + {}_nC_r x^{n-r}y^r + \cdots + nxy^{n-1} + y^n$.

Skills:

1. Calculate the binomial coefficients using the formula

   $${}_nC_r = \binom{n}{r} = \frac{n!}{(n - r)!\, r!}$$

   or by using Pascal's Triangle.

2. Expand a binomial.

   Example: Expand $(3x - 2y)^4$.

   Using Pascal's Triangle for $n = 4$, the coefficients are 1, 4, 6, 4, 1.

   Using the theorem pattern, decrease powers of $3x$ and increase powers of $2y$. The expansion is $(3x - 2y)^4 = (1)(3x)^4 - (4)(3x)^3(2y) + (6)(3x)^2(2y)^2 - 4(3x)(2y)^3 + (1)(2y)^4$.

Counting Principles

Fundamental Counting Principle:

If event E_1 can occur in m_1 different ways and following E_1, event E_2 can occur in m_2 different ways, then the number of ways the two events can occur is $m_1 \cdot m_2$.

Permutations: (order is important)

The number of permutations (orderings) of n elements is ${}_nP_n = n! = n(n - 1)(n - 2) \cdots 3 \cdot 2 \cdot 1$.

The number of permutations of n elements taken r at a time is

${}_nP_r = n!/[(n - r)!] = n(n - 1)(n - 2) \cdots (n - r + 1)$.

Combinations: (order is not important)

The number of combinations of n elements taken r at a time is

${}_nC_r = n!/[(n - r)!r!]$.

Probability

Probability of Event E

$$P(E) = \frac{n(E)}{n(S)}$$

where event E has $n(E)$ equally likely outcomes and sample space S has $n(S)$ equally likely outcomes.

Probability Formulas

$P(A \text{ or } B) = P(A \cup B)$

$P(A \text{ and } B) = P(A \cap B)$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$P(A \cup B) = P(A) + P(B)$, if A and B have no outcomes in common.

$P(A \cap B) = P(A) \cdot P(B)$, if A and B are independent events.

$P(\text{complement of } A) = P(A') = 1 - P(A)$

Study Capsule 7 Conics and Parametric Equations

Conics

Definitions

Circle: Locus of points equidistant from a fixed point, the center

Parabola: Locus of points equidistant from a fixed point (its focus) and a fixed line (the directrix)

Ellipse: Locus of points, so that the *sum* of their distances from two fixed points (foci) is constant

Hyperbola: Locus of points, so that the *difference* of their distances from two fixed points (foci) is constant

Eccentricity: $e = \frac{c}{a}$

parabola: $e = 1$

ellipse $0 < e < 1$

hyperbola $e > 1$

Standard Equations

Circle:

$(x - h)^2 + (y - k)^2 = r^2$

with center (h, k) and radius $= r$

Parabola:

$(x - h)^2 = 4p(y - k)$

$(y - k)^2 = 4p(x - h)$

with vertex (h, k) and $p =$ distance from the vertex to the focus

Ellipse:

$$\frac{(x - h)^2}{a^2} + \frac{(y - k)^2}{b^2} = 1$$

$$\frac{(x - h)^2}{b^2} + \frac{(y - k)^2}{a^2} = 1$$

with major axis $2a$, minor axis $2b$, distance from the center to focus is c, and $c^2 = a^2 - b^2$

Hyperbola:

$$\frac{(x - h)^2}{a^2} - \frac{(y - k)^2}{b^2} = 1$$

$$\frac{(y - k)^2}{a^2} - \frac{(x - h)^2}{b^2} = 1$$

$c^2 = a^2 + b^2$

General Equation:

$Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$

Basic Problem Types

1. Given information needed to find the parts (center, radius, vertices, foci, etc.), write the standard equation for the specified conic.
2. Given the general equation of a conic, complete the square and find the required parts of the conic. Then sketch its graph.
3. Given a general quadratic equation, use the coefficients A and C to classify the conic.

   Circle, if $A = C$

   Parabola, if $AC = 0$, A and C not both 0.

   Ellipse, if $AC > 0$

   Hyperbola, if $AC < 0$

Parametric Equations

Definitions

Definition: Parametric equations are used where the coordinates x and y are each a function of a third variable, called a *parameter*. Common parameters are time t and angle θ.

Standard Equations

Plane Curve C: If f and g are continuous functions of t on an interval I, the set of ordered pairs $(x(t), y(t))$ is a *plane curve* C. The equations $x = f(t)$ and $y = g(t)$ are *parametric equations* for C.

Basic Problem Types

1. Given the parametric equations for a plane curve C, construct a three-row table of values using input for t. Plot the resulting (x, y) points and sketch curve C. Then identify the orientation of the curve.
2. Given a set of parametric equations, eliminate the parameter and write the corresponding rectangular equation.
3. Given a rectangular equation, find a corresponding set of parametric equations using an appropriate parameter.

Index of Applications

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